

# Basic Ideas of Information Thermodynamics<sup>#</sup>

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**Abstract:** We apply a certain unifying physical description of the results of Information Theory. Assuming that heat entropy is a thermodynamic realization of information entropy, we construct a cyclical, thermodynamic, average-value model of an information transfer chain as a general heat engine, in particular a Carnot engine, reversible or irreversible. A working medium of the cycle (a thermodynamic system transforming input heat energy) can be considered as a thermodynamic, average-value model or, as such, as a realization of an information transfer channel. We show that for a model realized in this way the *extended II. Principle of Thermodynamics* is valid and we formulate its *information form*.

In addition we solve the problem of a proof of II. Principle of Thermodynamics. We state the relation between the term of information entropy, introduced by C. Shannon (1948), and thermodynamic entropy, introduced by R. Clausius (1850) and, further, explain the Gibbs paradox. Our way to deal with the given topic is a connection of both the mathematical definitions of information entropies and their mutual relations within a system of stochastic quantities, especially with thermodynamic entropies defined on an isolated system in which a realization of our (repeatable) observation is performed [it is a (cyclic) transformation of heat energy of an *observed, measured* system].

We use the *information* description to analyze the Gibbs paradox, reasoning it as a *property* of such *observation, measuring* an (equilibrium) thermodynamic system. We state a *logical* proof of the II. P.T. as a derivation of relations among the entropies of a system of stochastic variables, realized physically, and, the *Equivalence Principle* of the I., II. and III. Principle of Thermodynamics is formulated.

**Keywords:** Carnot cycle, I., II., III. Principle of Thermodynamics, Heat entropy, Observation, Information entropy, Transfer channel, Transinformation, Noise.

## 1. INTRODUCTION

A working medium of any heat cycle can be considered as a thermodynamic, average-value model or, as such, as a realization of an information transfer channel. More generally we could use some kind of *directly shared energy* [6]. The cycle models an information transfer process in the channel, which enables input messages to be transferred *repeatedly* and uses transformations of such energies.

Our thermodynamic-information derivation based on a heat cycle demonstrates the fact that it is impossible, in such a type of channel, for the *bound* [2] information contained in an input message to be transferred without its (average) loss, *even when the ideal case of a noiseless channel* is considered. This loss of information is the *necessary* condition for such a repeatable transfer of messages. Such information transfer can be worsened only by heat dissipation of energies, which means by a *noise* heat generated by the irreversible processes in the channel, *subtractive* in this case. This channel is described by a transformer of

input heat, which has non-ideal properties (inner friction).<sup>1</sup>

To explain the Gibbs paradox and to prove the II. P.T. we use the concept of *bound* information [2, 20]. This method is identical with introducing the *Boltzman* function of statistical physics. Its negative value, determined by a *detailness* of our description of an observed system, is proved to be a value of Clausius entropy (in a certain substitute equivalent equilibrium thermodynamic way [24]). We show that a physical realization of such observation is equivalent to a scheme of a relevant (reversible) heat cycle [10]. Its properties are expressible in terms of the Gibbs paradox.

The bound entropies of our realized observation, the input, the output and the conditional are, as the *free* ones [2, 20], associated by the *channel equation* [3, 26]. This equation is, in this sense, an *information* description of a cyclical transformation of heat energy of an observed, measured system [10]. In this way, its derivation proves the II. P.T. and gives its information forms as well [10]. It is a *most general* formulation of the II. P.T. and yields in the *Equivalence Principle* of the I., II. and III. P.T.

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<sup>#</sup>Supported by the grant of Ministry of Education of Czech Republic MSM 6046137307.

<sup>1</sup>This paper is a composite of the papers [10, 13] with small additions, and also, it is the very core of the book [14].

## 2. NOTION OF ENTROPY

### 2.1. Information Entropy

With a stochastic phenomenon  $\xi$ , the probability of which is  $p(\xi)$ , its "own" information, information amount  $I \stackrel{\text{Def}}{=} -K \cdot \log_z p(\xi)$  is associated. The quantity  $I$  is measured in information units *bit* when  $K=1, z=2$  or *nat* when  $K=1, z=e$  or *Hartley* when  $K=1, z=10$  is used. Also it is possible to measure it in *thermodynamic* units: *Boltzman* when  $K=k, k$  is *Boltzman constant*<sup>2</sup> and  $z=10$ ; for  $K=k$  and  $z=e$  it is *Clausius*. Then it is valid that  $k \cdot \text{Hartley} = \text{Boltzman}$  and  $k \cdot \text{nat} = \text{Clausius}$ .

Let the stochastic phenomenon  $\xi \in X$  be a realization of a discrete stochastic quantity  $X$  with a probability distribution  $[p(\cdot)]_X$ ,  $X$  is its selecting space,

$$X \stackrel{\text{Def}}{=} [X, p(\cdot)], \xi \in X, \sum_{\xi \in X} p(\xi) = 1, 0 \leq p(\xi) \leq 1. \quad (2.1)$$

The expectation value  $H(X)$  of the stochastic quantity  $[X, p(\cdot)]$ ,

$$H(X) \stackrel{\text{Def}}{=} E[X, p(\cdot)], \quad (2.2)$$

is called *Shannon, (information) entropy* [3] of  $X$ . With the denotation  $\xi \triangleq \xi_j$  and  $p(\xi_j) \triangleq p_j$ , we write

$$H(X) = -K \cdot \sum_j p_j \log_z p_j, \sum_j p(j) = 1. \quad (2.3)$$

For a continuous stochastic quantity  $X$  the differential entropy is defined [20],

$$H(X) \stackrel{\text{Def}}{=} -K \cdot \int_X w(\xi) \log_z w(\xi) d\xi; \int_X w(\xi) d\xi = 1, w(\xi) \geq 0; \quad (2.4)$$

$X \subseteq \mathfrak{R}$  is a non-degenerated interval,  $w(\xi)$  is a density of probability of  $X, \xi \in X$ .

#### 2.1.1. Channels of Information Theory

A discrete transfer channel  $K$  is defined as the tripartite structure

$$K \stackrel{\text{Def}}{=} [(X, p(\cdot|\cdot), Y)], \text{ where,} \quad (2.5)$$

$X$  is an *input* discrete stochastic quantity, a source of messages, with its selecting space  $X = \{x_1, x_2, \dots\}$  and a probability distribution  $[p(\cdot)]_X \stackrel{\Delta}{=} [p(\cdot)]_{\{i \in \{1,2,\dots\}\}}$ ;

$Y$  is an *output* discrete stochastic quantity, a receiver of messages. Its selecting space is  $Y = \{y_1, y_2, \dots\}$  and its probability distribution is  $[p(\cdot)]_Y \stackrel{\Delta}{=} [p(j)]_{\{j \in \{1,2,\dots\}\}}$ ;

the quantity  $p(\cdot|\cdot)$  is a probability distribution of an error in channel  $K$ , the distribution of probability of receiving (measuring, observing)  $y_j \in Y$  on the channel output when the input  $x_i \in X$  has been sent (output  $j$  for input  $i$ ).

Shannon entropy of  $X$  or  $Y$  respectively, is called the *input* or the *output entropy*, and, following the definition (2.3) with  $K=1$  and  $z=e$ ,

$$H(X) = -\sum_i q(i) \ln q(i), H(Y) = -\sum_j p(j) \ln p(j). \quad (2.6)$$

The information amount  $I_{ij}$ , contained in a phenomenon with the conditional probability  $p(i|j)$ , is defined by the equality  $I_{ij} \stackrel{\text{Def}}{=} -\ln p(i|j)$ , and, is called the *loss information*. For its average information, its (conditional) entropy, we figure:

$$H(X = x_i | Y) = -\sum_j p(i|j) \ln p(i|j), \quad (2.7)$$

$$H(X|Y) = \sum_i q(i) \sum_j p(i|j) \ln p(i|j) = -\sum_i \sum_j p(i,j) \ln p(i|j),$$

where  $p(i,j)$  is the simultaneous probability of a phenomenon  $i$  and  $j$ . The conditional entropy  $H(X|Y)$  is called the *loss (residual) entropy*.

Similarly,  $I_{ji} \stackrel{\text{Def}}{=} \ln p(j|i)$ . This quantity is called the *noise information*. Its average information, the conditional entropy, is called the *noise entropy* in channel  $K$ ,

$$H(Y|X) = -\sum_i \sum_j p(i,j) \ln p(j|i). \quad (2.8)$$

The usable information  $I_{i,j}$  in an output message  $y_j$  about an input  $x_i$  is defined by the equality  $I_{i,j} \stackrel{\text{Def}}{=} I_i - I_{ij} = -\ln q(i) + \ln p(i|j) = \ln \frac{p(i|j)}{q(i)}$ . The

average information amount of  $I_{i,j}$  in the message  $j$  about the message  $i$  is the quantity

<sup>2</sup>Gay-Lussac experiment,  $k = \frac{R}{N_A}$ ,  $R$  is gas constant,  $N_A$  is Avogadro number [15]

$$T(X;Y) \stackrel{\text{Def}}{=} \sum_i \sum_j I_{i;j} p(i,j) = \sum_i \sum_j p(i,j) \ln \frac{p(i|j)}{q(i)} \quad (2.9)$$

$$= -H(X|Y) + H(X),$$

called the average usable *transferred information, transinformation* [20]. Similarly,

$$I_{j;i} = I_j - I_{ji} = -\ln p(j) + \ln p(j|i) = \ln \frac{p(j|i)}{p(j)},$$

and then

$$T(Y;X) \stackrel{\text{Def}}{=} \sum_i \sum_j I_{j;i} p(i,j) = \sum_i \sum_j p(i,j) \ln \frac{p(j|i)}{p(j)} \quad (2.10)$$

$$= -H(Y|X) + H(Y).$$

For a simultaneous distribution  $p(i,j) [= p(j,i)], i, j \in \{1, 2, \dots\}$  we define the *simultaneous entropies*

$$H(X,Y) \stackrel{\text{Def}}{=} -\sum_i \sum_j q(i)p(j|i) \ln[q(i)p(j|i)] \quad (2.11)$$

$$= H(X) + H(Y|X),$$

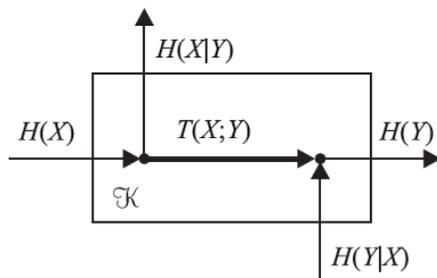
$$H(Y,X) \stackrel{\text{Def}}{=} -\sum_i \sum_j p(j)p(i|j) \ln[p(j)p(i|j)]$$

$$= H(Y) + H(X|Y)$$

and then  $H(Y,X) = H(X,Y)$ .

A *continuous transfer channel K* is defined in a similar formal way as a discrete one, but the quantities  $X, Y, (X|Y), (Y|X)$  are continuous. Adequate differential entropies are defined [20].

It follows from equalities (2.9)-(2.11) that, both in a discrete and in a continuous transfer channel, the *law of entropy (information) conservation, the channel equation* is valid,

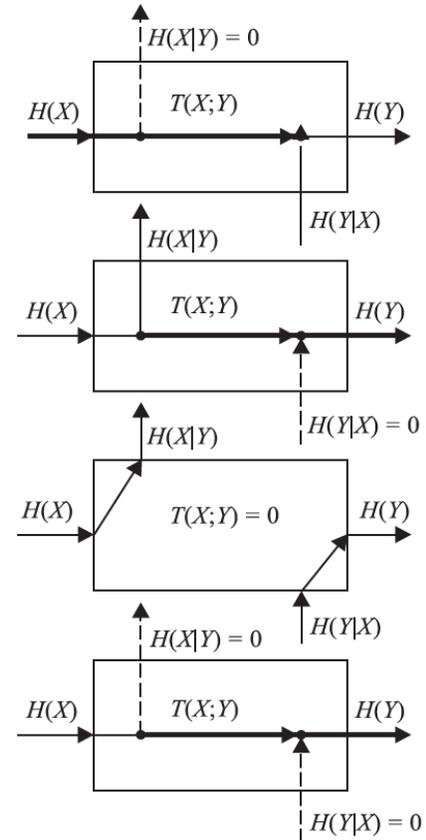


**Figure 1:** The relations among entropies of an information transfer channel.

$$H(X) + H(Y|X) = H(Y) + H(X|Y), \quad H(X) - H(X|Y) \quad (2.12)$$

$$= H(Y) - H(Y|X).$$

Or, also, the *symmetry* of transformation,  $T(X;Y) = T(Y;X)$ , is valid.



**Figure 2:** The information transfer channel: without losses, without noise, disrupted, without noise and losses.

## 2.2. Thermodynamic Entropy

### 2.2.1. Definition of Equilibrium Macroscopic Thermodynamics

With an arbitrary thermodynamic system  $A$ , in the *thermodynamic equilibrium*, the *macroscopic (global, extensive)* and thus the *additive* quantity, called the *thermodynamic (Clausius) entropy*, denoted as  $S$ , is associated. But, the *phenomenologic (macroscopic, classic)* thermodynamics defines its change  $\Delta S$  only, generated by a reversible exchange of heat  $\Delta q$  at an *absolute* constant temperature (in  $^\circ\text{K}$ ) between the system  $A$  and its *environment*. Or, it should be defined by another change of heat energy  $\Delta q$ , expressible by an equivalent *substitute* reversible way<sup>3</sup> at a certain constant temperature  $\Theta > 0$  [24]. This change is defined by the *Clausius equation*

<sup>3</sup>or, by a possibility of such a type of changes

$$\Delta S \stackrel{\text{Def}}{=} \frac{\Delta q}{\Theta}, \Theta > 0. \tag{2.13}$$

By integration of  $dS \stackrel{\Delta}{=} \Delta S$ , when  $\delta q \stackrel{\Delta}{=} \Delta Q$ , the entropy  $S$  can be figured, but, with such an exception of an additive integration constant  $S_0$ . For the entropy  $S$ , as a function of a temperature  $\Theta > 0$ , it is to be valid that

$$S = S(\Theta) = \int \frac{\delta q}{\Theta} = \sigma(\Theta) + S_0, \tag{2.14}$$

where  $\Theta^{-1}$  is Pfaffian integrating factor [5, 21].

The quantity  $S_0$  is a non-zero constant, independent from *state variables* of system  $A$ , but dependent on *matter units* (*mol, kmol*) of  $A$ . Then,  $S_0 \stackrel{\Delta}{=} S_0(n)$ , where  $n$  is a number of matter units of  $A$ . Not respecting this fact leads to the known *Gibbs paradox* [17, 19, 23]. A certain *mathematical* solution of this phenomenon will now be revised shortly.

### 2.2.2. Gibbs Paradox

Just by the (thought) "dividing" of an equilibrium system  $A$  by *diaphragmas* [23], without any influence on its thermodynamic (macroscopic) properties, a non-zero difference of its entropy, before and after its "dividing", is evidenced.

Let us consider a thermodynamic system  $A$  in volume  $V$  and with  $n$  matter units of ideal gas in the thermodynamic equilibrium. The *state equation* of  $A$  is  $pV = nR\Theta$ . For an elementary change of the *internal energy*  $U$  of  $A$  we have  $dU = nc_v d\Theta$ .

From the state equation of  $A$  and from the general *law of energy conservation* [for a (substitute) reversible exchange of heat  $\delta q$  between the system and its environment] we formulate the *I. Principle of Thermodynamics*,  $\delta q = dU + pdV$ .

From this principle, and, from the Clausius equation (12), it follows that

$$S = n \int \left( c_v \frac{d\Theta}{\Theta} + R \frac{dV}{V} \right) = n(c_v \ln \Theta + R \ln V) + S_0(n) = \sigma(\Theta, V) + S_0(n). \tag{2.15}$$

Let us "divide" the equilibrial system  $A$  in a volume  $V$  and at a temperature  $\Theta$ , or, better said, the whole volume  $V$  (or, its whole given state space) occupiable, and now just occupied by all its constituents (particles, matter units), with diaphragmas (thin infinitely, or, "thought" only), not affecting the thermodynamic properties of  $A$  supposingly, to  $m$  parts  $A_i$ ,

$i \in \{1, \dots, m\}$ ,  $m \geq 1$  with volumes  $V_i$  with matter units  $n_i$ . Evidently,  $n = \sum_{i=1}^m n_i$  and  $V = \sum_{i=1}^m V_i$ .

Now let  $S_0(n) = 0$  and  $S_{0i}(n_i) = 0$  for all  $i$ . For the entropies  $S_i$  of  $A_i$  considered individually, and for the change  $\Delta S$ , when volumes  $V, V_i$  are expressed from the state equations, and for  $p = p_i$ ,  $\Theta = \Theta_i$  it will be gained that  $\sigma_{[i]} = Rn_{[i]} \ln n_{[i]}$ . Then, for  $S_i = \sigma_i = n_i(c_v \ln \Theta + R \ln V_i)$  is to be valid, we have

$$\sum_{i=1}^m S_i = \sum_{i=1}^m \sigma_i = nc_v \ln \Theta + R \ln \left( \prod_{i=1}^m V_i^{n_i} \right), \tag{2.16}$$

$$\begin{aligned} \Delta S = S - \sum_{i=1}^m S_i &= \sigma - \sum_{i=1}^m \sigma_i = \Delta \sigma = R \ln \frac{V^n}{\prod_{i=1}^m V_i^{n_i}} \\ &= -nR \sum_{i=1}^m \frac{n_i}{n} \ln \frac{n_i}{n} > 0. \end{aligned}$$

Let us denote the last sum as  $B$  further,  $B < 0$ .

The result (2.16),  $\Delta S = -nRB$ , is a *paradox*, a contradiction with our presumption of not influencing a thermodynamic state of  $A$  by diaphragmas, and, leads to that result, that the heat entropy  $S$  (of a system in equilibrium) is *not* an extensive quantity. But, by the definition of the differential  $dS$ , this *is not* true.

Due to this contradiction<sup>4</sup> we must consider non-zero integrating constants  $S_0(n)$ ,  $S_{0i}(n_i)$ , in such a way, that  $\Delta S = (\sigma + S_0) - \sum_{i=1}^m (\sigma_i + S_{0i}) = 0$ . This equation is solvable for the system  $A$  and all its parts  $A_i$  by solutions  $S_{0[i]}(n_{[i]}) = -n_{[i]} R \ln \frac{n_{[i]}}{\gamma_{[i]}}$ .

Then  $S_{[i]} \stackrel{\Delta}{=} S_{\text{Claus.}[i]}$ , and we write and derive that

$$\begin{aligned} S_{\text{Claus.}} &= \sum_{i=1}^m S_{\text{Claus.}i} = \sum_{i=1}^m n_i R \ln \gamma_i = nR \ln \gamma \\ &\Rightarrow \gamma = \gamma_i; \Delta S = 0. \end{aligned} \tag{2.17}$$

<sup>4</sup>Quantity  $-B$  expressed in (2.16) is information entropy of a source of messages with an alphabet  $[n_1, n_2, \dots, n_m]$  and probability distribution  $\left[ \frac{n_i}{n} \right]_{i=1}^m$ . Such a division of system to  $m$  parts defines an information source with information entropy of maximum  $\ln m$ .

### 2.2.3. Definition of Statistical Thermodynamics

The term (*macro*)state of a system  $A$ , both *equilibrium* and *non-equilibrium*, means a *class of equivalence*<sup>5</sup> on the set of all its possible *microstates* - microscopic *arrangements* of all its constituents (particles, matter units) within all (by us distinguished, defined) parts, *cells*, of its whole, by them occupiable, volume  $V$ , or, better said, of a given (*state*) space of  $A$ <sup>6</sup>.

Let us denote the cardinality of such a class as *thermodynamic probability*  $\tilde{P}$  of a given macrostate.

Let  $m$  be a number of cells of the whole volume, (*state*) space, of  $A$ , occupiable by a set of all its constituents, let  $M$  be a number of all its constituents, and, let  $m_l$ ,  $0 \leq m_l \leq M$ , be a number of constituents in a cell  $l$  of a given (*state*) space,  $1 \leq l \leq m$ . Then,

$$\tilde{P} = \frac{M!}{\prod_{l=1}^m m_l!}, \quad \sum_{l=1}^m m_l = M.$$

Then the *Boltzman definition* [15] of the physical (*thermodynamic*) entropy  $S$  of  $A$  (of, the whole volume  $V$ , or, of a given state space) in a macrostate with thermodynamic probability  $\tilde{P}$  [could be non-equilibrium too, composed by (*equilibrium*) subsystems  $A_i$  in states  $\theta_i$ ,  $i \in \{1, 2, \dots, m\}$ , not interacting mutually (at the same temperature  $\Theta$ , or also, at various temperatures  $\Theta_i$ ) with thermodynamic probabilities  $\tilde{P}_i$ ] is valid;

$$S_{\text{Boltz}} \stackrel{\text{Def}}{=} k \cdot \ln \tilde{P}, \quad S_{\text{Boltz}} = \sum_{i=1}^m S_{\text{Boltz},i} = k \cdot \sum_{i=1}^m \ln \tilde{P}_i, \quad \tilde{P} = \prod_{i=1}^m \tilde{P}_i. \quad (2.18)$$

Let us denote by  $N$  and  $N_j$  numbers of particles of system  $A$  and of its subsystems  $A_j$ ,  $j = 1, \dots, m \leq N$ ,

$$N_j = n_j N_A, \quad \sum_j N_j = N = n N_A.$$

By Stirling's formula and (2.18) we have, that

$$S_{\text{Boltz}} = -kN \sum_j \frac{N_j}{N} \ln \frac{N_j}{N} = -kNB.$$

The last sum  $B$  defines *Boltzman function*  $B_{\text{Boltz}}$  of statistical physics,  $B^{\text{Boltz}} = B$ .

Evidently,  $\ln \tilde{P} = -NB_{\text{Boltz}}$ .

For Boltzman entropy per one particle we write

$$\frac{S_{\text{Boltz}}}{N} = -kB_{\text{Boltz}}.$$

The equilibrium system  $A$  (with  $N$  particles) is possible to be identified with the (*state*) space it occupies, being defined by "one-particle" railing of cells ["*one-particle*" (*state*) space], with the uniform probability distribution of occupation these cells by particles,  $\left[\frac{N_j}{N}\right]_j = \left[\frac{1}{N}\right]_j$ , where  $j = 1, \dots, m$  and  $m = N$ .

In this space the quantities  $B^*$ ,  $\tilde{P}^*$ ,  $S^*$  are defined,

$$B^* = \sum_{j=1}^N \frac{1}{N} \ln \frac{1}{N} = \ln N = -\frac{1}{N} \ln \tilde{P}^*, \quad S^* = -kNB^*. \quad (2.19)$$

### 3. CLAUSIUS, BOLTZMAN AND SHANNON ENTROPY

Let us explore the entropy  $S_{\text{Boltz}}$  during the Gay-Lussac experiment.

The value  $S_{\text{Boltz}}$  is dependent on a time development of  $A$  within the whole volume  $V$  divided into cells, and, in this level of detail  $S^* = \max_{\{m,p(\cdot)\}} \{S_{\text{Boltz}}\}$ . In the final state of  $A$  (*equilibrium*) we have:  $\Theta = \text{const.}$ ,  $n = \frac{N}{N_A}$  and  $n_i = \frac{N_i}{N_A}$ , where  $i = 1, 2, \dots, m$  is given by a railing of our diaphragmas.

Probability distribution  $\frac{N_i}{N}$ ,  $i = 1, \dots, m$  (defined in such railing) states a probability of an *observer's choice* of  $i$ -th cell with  $N_i$  particles.

The uncertainty  $-B$  of this choice is given by Shannon entropy at the value

$$-B_{\text{Boltz}} = -\sum_{i=1}^m \frac{N_i}{N} \ln \frac{N_i}{N} = -B_{\text{Gibbs}}.$$

Its maximum  $-B^*$  is achieved for the most possible detailed description ( $m = N$ ) of the observed equilibrium system  $A$  and with the uniform probability distribution  $\frac{N_i}{N} = \frac{1}{N}$ ,  $i = 1, \dots, N$ .

Till this moment, the expression  $-B_{\text{Boltz}}$  (2.18) defines an uncertainty of probability distribution too, but, defined by a *time propagation* of the set of particles of  $A$ , now not equilibrium, within the all cells of the given railing.

<sup>5</sup>Due to individual particles being undistinguishable mutually by means of macroscopic thermodynamics, microstates of a given macrostate are undistinguishable too

<sup>6</sup>State space of  $A$  can be *three-dimensional* or *general configuration space*, *impulse* or the whole *phase space*

When  $A$  achieves its final, equilibrium state, each "its own" "one-particle" cell is occupied by one particle and the thermodynamic entropy of  $A$  (of its entire volume, or, of a given state space) per one cell, is  $-kB^*$ .

But when the observer's railing is considered, the index  $i = j = 1, \dots, m$  is used, and, the value  $-B_{\text{Boltz}} = -B_{\text{Gibbs}} \leq B^*$  is defined. Till this moment, the relation  $-B_{\text{Boltz}} < -B_{\text{Gibbs}} \leq -B^*$  is valid.

During the time development of  $A$  such probability distributions are defined that are impossible for observing its equilibrium state (for which our railing defines the value  $-B_{\text{Gibbs}}$ ); for the sets of all values of  $-B_{\text{Gibbs}}$  and  $-B_{\text{Boltz}}$  we have

$$\{-B_{\text{Gibbs}}\} \subset \{-B_{\text{Boltz}}\}, \text{ but, } \max\{-B_{\text{Gibbs}}\} = \max\{-B_{\text{Boltz}}\} \\ = -B^* = \frac{S^*}{kN}.$$

The quantity  $-B^*$ , introduced in (2.19), has a meaning of the maximal uncertainty of distribution of particles, and, also, of their energies within all "one-particle" cells.

Thus, using a measure  $r$  of the accuracy, detail of our observation, we write

$$-B_{\text{Gibbs}} = -\sum_{i=1}^m \frac{N_i}{N} \ln \frac{N_i}{N} = -\frac{1}{r} \sum_{j=1}^N \frac{1}{N} \ln \frac{1}{N} \\ = \frac{1}{r} \ln N = -\frac{1}{r} B^*, \quad r \geq 1, \tag{3.1}$$

$$\ln N = -r \sum_{i=1}^N \frac{N_i}{N} \ln \frac{N_i}{N} = -r \cdot B_{\text{Gibbs}} = -r \cdot B_{\text{Boltz}} = -B^*,$$

$$S^* = kN \ln N = -kNB^* = S_{\text{Claus}} = -r \cdot kNB = rS_{\text{Gibbs}} = rS_{\text{Boltz}}.$$

Following (3.1), for the system  $A$  with "its own" "one-particle" railing,  $j = 1, \dots, N$ , and, for an observer's railing,  $i = 1, \dots, m$ , it is valid that

$$\ln \gamma = -s \cdot B, \quad s = \frac{r \cdot Q}{\Theta \cdot kN \ln N}, \quad s > 0, \quad sr = \frac{\ln \gamma}{\ln N} = \text{const.} \tag{3.2}$$

It is provable that  $s = r$  for all  $r \geq 1$ , and thus,  $\gamma = N > 1$ .

For  $A$  with volume  $V$ , in the thermodynamic equilibrium at a temperature  $\Theta$ , its heat  $Q = \int_V \delta q(\Theta, V)$  and its entropy

$$S_{\text{Claus}} = \int_V \frac{\delta q(\Theta, V)}{\Theta} = \frac{Q}{\Theta}.$$

Then, following note 3, we have,

$$S_{\text{Claus}} = \frac{Q}{\Theta} = kN \ln \gamma = kN \ln N = S^*, \quad \gamma = \frac{\varepsilon}{e^{k\Theta}}, \quad \varepsilon = \frac{Q}{N} = \frac{Q_i}{N_i}. \tag{3.3}$$

For the (final) equilibrium state of  $A$  and for our most possible detailed observation of  $A$  [ $\max(i) = m = N$  with the uniform probability distribution of our choice]  $r = s = 1$  is valid. Otherwise  $s = r > 1$ .

Considering a general observer's railing,  $i = 1, \dots, m \leq N$ , any time development of  $A$  is expressed by a sequence of values from the set  $\{S_{\text{Boltz}}\} - \{S_{\text{Boltz}}\} \cap \{S_{\text{Gibbs}}\}$ , growing, defined by a time propagation of its  $N$  particles (bearing heat  $Q$  at temperature  $\Theta$ ) through its whole occupiable volume (a state space). Any of values of such a sequence defines Clausius entropy, relevant to a certain equivalent substitute equilibrium state, at value  $\frac{S_{\text{Claus}}}{r'} = \frac{Q}{T'} = \frac{Q}{r'\Theta}$ ,  $r' \geq r \geq 1$ .

On the set  $\{S_{\text{Boltz}}\} \cap \{S_{\text{Gibbs}}\}$  the equality  $-B_{\text{Boltz}} = -B_{\text{Gibbs}}$  is valid. It expresses the observer's uncertainty only (as  $r' = r$ ). It can be changed by a various organization of railing, up or down respectively. Thus, the best possible knowledge about the equilibrium  $A$  should be achieved just when both the observer and the system  $A$  are in equilibrium and "one-particle" railing is considered.

$$\text{Then, } r' = r = 1, \text{ otherwise, } \frac{S_{\text{Boltz}}}{S_{\text{Gibbs}}} = \frac{r}{r'} \leq 1.$$

The entropy  $S_{\text{Claus}}$  is the maximum of the physical entropies  $S_{\text{Boltz}}$  and  $S_{\text{Gibbs}}$  which are bound Shannon entropies figured in physical units only.

## 4. ELEMENTS OF HEAT TRANSFORMATION

### 4.1. Reversible Carnot Cycle

A reversible Carnot cycle, here denoted as  $O$ , is the most simple heat *cyclical process* ending with a gain of mechanical work, denoted as  $\Delta A$ . It consists of: 4 *reversible* changes; 2 isothermal changes related to two constant absolute temperatures,  $T_w$  and  $T_0$ ,  $T_w \geq T_0 > 0$ ; and two adiabatic changes effecting the transposition between these two temperatures in a *working medium*. The working medium, a *thermodynamic system* denoted as  $L$ , is the transformer of the input energy, denoted as  $\Delta Q_w$ , and as it passes through the cycle it acquires this *input* heat  $\Delta Q_w$ . This occurs during the reversible isothermal

expansion phase, during which the temperature  $T_w$  of the system is equal to that of the heater  $A$ . Within the reversible isothermal compression phase, while the temperature  $T_0$  of the system is equal to that of the cooler  $B$ , the system  $L$  is giving out "residual" heat, denoted as  $\Delta Q_0$ , to  $B$ . To achieve a positive gain in  $\Delta A$  it must be valid that  $T_w > T_0$ . The difference between the values of  $\Delta Q_w$  and  $\Delta Q_0$  is equal to that of  $\Delta A$  gained within the cycle;

$$\Delta A = \Delta Q_w + (-\Delta Q_0) \text{ or } \Delta A = \Delta Q_w + \Delta Q_0^*, \text{ when } \Delta Q_0^* = -\Delta Q_0.$$

The sign  $+$  denotes the heats delivered (directly) to the system  $L$  and the sign  $-$  denotes the heats drained off (directly) from the system  $L$  during the cycle.

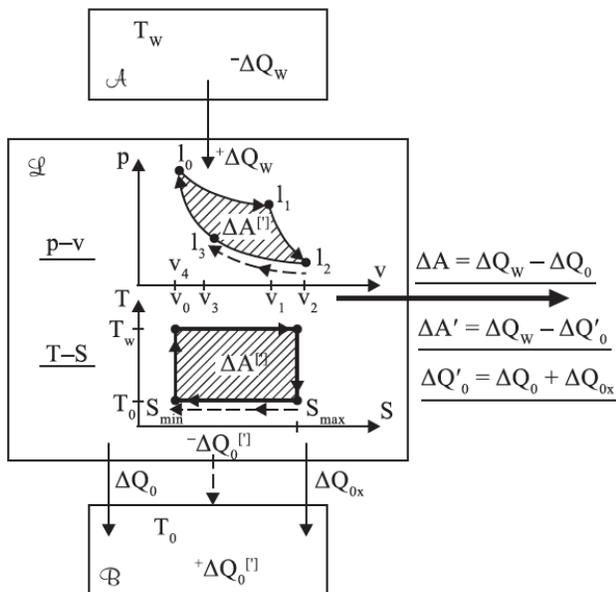


Figure 3: Carnot Cycle.

The transformation efficiency of a reversible Carnot cycle, denoted as  $\eta_{max}$ , is defined by the equation

$$\eta_{max}^{Def} = \frac{\Delta A}{\Delta Q_w} = \frac{\Delta Q_w + (-\Delta Q_0)}{\Delta Q_w}. \quad (4.1)$$

Following from both the definition (4.1) and the condition of the cyclical form of the whole thermodynamic change  $O$  [15], we have

$$\eta_{max} = \frac{\Delta Q_w - \Delta Q_0}{\Delta Q_w} = \frac{T_w - T_0}{T_w}, \eta_{max} < 1. \quad (4.2)$$

Carnot's theorem (see later) states that  $\eta_{max}$  is the maximum of a set of efficiencies from all those heat cycles with those (extreme) working temperatures. Following from (4.2) in a reversible Carnot cycle it is valid that

$$\frac{\Delta Q_w}{T_w} = \frac{\Delta Q_0}{T_0}, \frac{\Delta Q_w}{T_w} + \left(-\frac{\Delta Q_0}{T_0}\right) = 0 \text{ or } \frac{\Delta Q_w}{T_w} + \frac{\Delta Q_0^*}{T_0} = 0, \quad (4.3)$$

or more generally that

$$\sum_{i \in \{W,0\}} \frac{\Delta Q_i}{T_i} = \oint_o \frac{\delta Q(\Theta)}{\Theta} = 0. \quad (4.4)$$

The expressions  $\Delta Q_i$  and  $\delta Q(\Theta)$  denote all the heats shared reversibly (isothermally) when the temperature of the system  $L$  is either  $T_i$  or  $\Theta$  respectively. The ratios  $\frac{\Delta Q_i}{T_i}$  and  $\frac{\Delta Q(\Theta)}{\Theta}$  are known as changes of heat (Clausius') entropy  $S$ .

The integral in (4.4) is named Clausius' integral and this relation itself is the mathematical integral form of the II. Principle of Thermodynamics in Thomson-Planck's formulation: It is impossible to construct a heat cycle transforming all heat delivered to a system ( $L$ , going through this cycle) into an equivalent amount of mechanical work ( $\Delta A$ ).

Therefore in the case of a reversible Carnot cycle, Clausius' integral, the algebraic sum of all the by-temperature-reduced heats [both delivered (directly) to the cycle and drained off (directly) from the cycle], equals 0. According to Thomson-Planck's formulation there must be both heat  $\Delta Q_{i=w}$  delivered into the cycle  $O$  and heat  $\Delta Q_{i=0}$  drained off from the cycle. As a consequence of the existence of these heats (the input  $\Delta Q_w$  and the "residual"  $\Delta Q_0$ ), and in accordance with the I. Principle of Thermodynamics, it must be valid that  $\eta_{max} < 1$ .

Another formulation of the II. Principle of Thermodynamics is Carnot's theorem, the first part of which states: The efficiencies of all reversible Carnot cycles with the working temperatures  $T_w$  and  $T_0$  are equal.

A reversible Carnot cycle is an abstract construction working without any restriction on the values  $T_w$  and  $T_0$  (except in the case that  $T_w \geq T_0$ ) and any restriction on the medium  $L$ . Therefore we can consider that it runs in both an ideal medium and a non-ideal medium.

Reversibility of a (thermodynamic) change of status in a thermodynamic system means that, in such a system, there is no resistance that this change must overcome. Thus there is no friction in a system going through such a change, and consequently there is no (positive) production of heat within it. We can denote such heat as  $\Delta Q_{0,x}$ . This situation, where  $\Delta Q_{0,x} = 0$ , stands in any ideal medium  $L$  (without viscosity, i.e. in

ideal gas), both when a Carnot cycle runs finitely slowly and when it runs infinitely slowly. We can call this an ideal cycle.

However, a reversible Carnot cycle can run in a non-ideal, real system  $L$  (with viscosity) when the kinetic energy of the system is constant and, as a consequence, no heat  $\Delta Q_{0,x} > 0$  is generated. This situation occurs when the change of states in a (real) system is quasi-stationary, namely when it is infinitely small and runs infinitely slowly, so that it is valid that  $\Delta t \rightarrow \infty$  (where  $t$  denotes time). In this case  $\Delta Q_{0,x} = 0$  also holds true.

### 4.2. Irreversible Carnot Cycle

When the system  $L$  is non-ideal, for instance it is a real gas and the heat cycle operates with a finite, but not zero, speed,

$$0 < \Delta t < \infty, \tag{4.5}$$

some inner friction caused by the viscosity of  $L$  generates noise heat  $\Delta Q_{0,x}$ . The fact that the medium  $L$  is non-ideal is only discovered when the changes the system undergoes are not reversible (e.g. they are not the cycle  $O$  where  $\Delta t \rightarrow \infty, \Delta Q_{0,x} = 0$  is valid), namely when it undergoes non-quasi-stationary changes lasting a finitely long period of time (4.5). Then the kinetic energy of  $L$  varies and, due to inner friction on the passive resistants in  $L$ , (positive) production of the heat  $\Delta Q_{0,x}$  arises,

$$\Delta Q_{0,x} > 0. \tag{4.6}$$

The second part of Carnot's theorem states: The efficiency of any irreversible heat cycle with (extreme) working temperatures  $T_w$  and  $T_0$ ,  $T_w \geq T_0 > 0$ , is less than the efficiency of a reversible Carnot cycle with those same working temperatures.

The substitute thermodynamic path [24] is denoted as  $O'$ , expressing the irreversible cycle itself. For this path to be closed, the heat  $\Delta Q''_0$  must be drained off from the medium  $L$  when its temperature is  $T_0$ . This heat is greater than the heat  $\Delta Q_0$  when only the reversible cycle  $O$  is considered. Thus we have

$$\Delta Q''_0 = \Delta Q_0 + \Delta Q_{0,x}. \tag{4.7}$$

Consequently for the output work  $\Delta A'$  of an irreversible Carnot cycle we have

$$\Delta A' = \Delta Q_w + (-\Delta Q''_0) = \Delta A - \Delta Q_{0,x}. \tag{4.8}$$

For an irreversible Carnot cycle with the working temperatures  $T_w$  and  $T_0$ , where  $T_w \geq T_0 > 0$ , it follows

from the definition of the (transformation) efficiency, denoted as  $\eta$ , that

$$\eta = \frac{\Delta A'}{\Delta Q_w} < \frac{\Delta A}{\Delta Q_w} = \frac{T_w - T_0}{T_w} = \eta_{max}. \tag{4.9}$$

The equality (4.8) and the inequality (4.9) implies that

$$\frac{\Delta Q''_0}{\Delta Q_w} > \frac{T_0}{T_w}. \tag{4.10}$$

Within the reversible part  $O$  of the irreversible cycle  $O'$  the equality (4.3) is valid and thus, following (4.7) and (4.10), for the whole irreversible  $O'$  we have

$$\frac{\Delta Q_w}{\Delta T_w} + \left( -\frac{\Delta Q''_0}{T_0} \right) = -\frac{\Delta Q_{0,x}}{T_0} < 0, \tag{4.11}$$

or, more generally (Clausius' inequality),

$$\sum_{i \in \{W, O\}} \frac{\Delta Q(T_i)}{T_i} \stackrel{\Delta}{=} \oint_{O'} \frac{\delta Q(\Theta)}{\Theta} < 0, \quad \Delta Q(T_i) \stackrel{\Delta}{=} \Delta Q_i, \tag{4.12}$$

where  $\Delta Q_i$  and  $\delta Q(\Theta)$  denote all the (elementary) heats shared reversibly (isothermally) when the temperature of the system  $L$  is respectively either  $T_i$  or  $\Theta$ . The relations (4.4) and (4.12) are also known as Kelvin's formulation of the II. Principle of Thermodynamics.

In the case of an irreversible Carnot cycle  $O'$ , Clausius' integral, the algebraic sum of all the by-temperature-reduced heats (4.12) [both delivered (directly) to the cycle and drained off (directly) from the cycle], is less than 0. This is caused by the heat  $\Delta Q_{0,x}$  being drained off from the medium  $L$  (into  $B$ ) when its temperature is  $T_0$ . This is a consequence of the requirement that the thermodynamic path  $O'$  in  $L$  must be cyclical, and, thus, that the whole process be repeatable.

Thus with respect to the work  $\Delta A'$  produced within one run of an irreversible Carnot cycle  $O'$ , this cycle can be considered to be a reversible cycle  $O$  accompanied by the generation of noise heat  $\Delta Q_{0,x} > 0$ . The "additional" heat  $\Delta Q_{0,x}$  existing in an irreversible cycle then enters  $B$  during the compression phase, with the compression work having a value equivalent to the noise heat  $\Delta Q_{0,x}$ . This compression work diminishes the mechanical work  $\Delta A$  acquired during the reversible part  $O$  of the  $O'$  cycle. Thus we can envisage one "additional" isothermal compression running in parallel with the "original" compression. Therefore, with regards to the heat  $\Delta Q''_0$  being drained

off from the medium  $L$ , when  $L$  is going through the irreversible cycle  $O'$ , this cycle can be considered to be an *additive* superposition of the reversible part  $O$  and the irreversible part; the latter simply being this "additional" isothermal compression.

*Irreversibility* is the consequence of both the medium  $L$  being non-ideal and its state changes being of finite and non-zero speed. Therefore the equation  $\Delta Q_w = \Delta A = \Delta A'$  is the limit, but *not achievable* in reality.

**4.3. Reverse Reversible Carnot Cycle**

This cycle is a reverse Carnot cycle in which the states of the system  $L$  are changing along the curve  $O$ , but, in the opposite way, starting with the isothermal expansion at the temperature  $T_0$  when the *diathermal* contact between  $L$  and  $B$  is maintained. It is a cycle run of the medium  $L$  through a sequence of (equilibrium) states along the curve  $O$  oriented in the opposite way than it is in the case of a direct Carnot cycle. This run is called the *cooling* run too. The Carnot machine with this running through of its medium  $L$  along the heat cycle  $O$  is working as a *heat pump, cooling machine*. The functionality of such a cooling machine is that:

Within the isothermal expansion at the temperature  $T_0$  the system  $L$  is draining off the heat  $\Delta Q_0$  being *pumped out, transferred* from the cooler  $B$ . This heat is, within the isothermal compression at the temperature  $T_w$ , delivered from  $L$  into the heater  $A$ . We call it the *output* heat  $\Delta Q_w$ ,

$$\Delta Q_w = \Delta Q_0 + \Delta A \tag{4.13}$$

The quantity  $\Delta A$  is the *input* mechanical energy (work) delivered into  $L$  within the isothermal compression at the temperature  $T_w$ .

Following the first part of Carnot's theorem (the efficiencies of all reversible Carnot cycles with the working temperatures  $T_w$  and  $T_0$ ,  $T_w \geq T_0 > 0$ , are equal) we must conclude that in the opposite case we would be able to couple two Carnot machines with different efficiencies  $\eta_1, \eta_2, \eta_1 \neq \eta_2$  in such a way that the resulting machine would be the *perpetuum mobile* of the II. order (machine changing cyclically, permanently the whole input heat  $\Delta Q_w$  in the output work  $\Delta A$ ;  $\Delta Q_w = \Delta A$ ). So it must be valid that  $\eta_1 = \eta_2$ .

The first part of Carnot's theorem is another variant of the Thomson-Planck formulation of the II. Principle of Thermodynamics.

**5. HEAT TRANSFORMATION AND INFORMATION TRANSFER**

It follows from section 3 and also from Brillouin [2] and [20], Landauer [22] and Gershenfeld [4] that when an average amount of information (denoted as  $\Delta J$ ) is being *recorded, transmitted* or *computed*, etc. while the temperature is  $\Theta$ , there is a need for a (minimal) average energy  $\Delta W$ ,  $\Delta W \geq k \cdot \Theta \cdot \Delta J$ . In this paper  $\Delta W \stackrel{\Delta}{=} \Delta Q_w$ .

We term the following ratios [expressed in information units (*Hartley, nat, bit*)] changes of the thermodynamic entropies of system  $L$  in cycles  $O$  or  $O'$ :

$$\frac{\Delta Q_w}{kT_w} \text{ input}, \frac{\Delta A^{[I]}}{kT_w} \text{ output } (= \Delta I^{[I]}), \frac{\Delta Q_0}{kT_w} \text{ loss}, \frac{\Delta Q_{0x}}{kT_w} \text{ noise}, \tag{5.1}$$

where  $k$  is Boltzman's constant.

These changes are the absolute values of  $H(\cdot)$ ,  $H(\cdot|\cdot)$  contained in any message on inputs and outputs of a "carnot" (*thermodynamically*) described transfer channel  $K$ .

**5.1. Reversible Carnot Cycle and Noiseless Channel**

A reversible Carnot cycle  $O$  running in  $L$  (producing noise heat  $\Delta Q_{0x} = 0$ ) can be considered to be a *thermodynamic, average-value* realization or, as such, as a model of an information transfer process running in a channel  $K$  *without noise*. For the average noise information (entropy)  $H(Y|X)$  defined in (2.8) it is valid that  $H(Y|X) = 0$ .

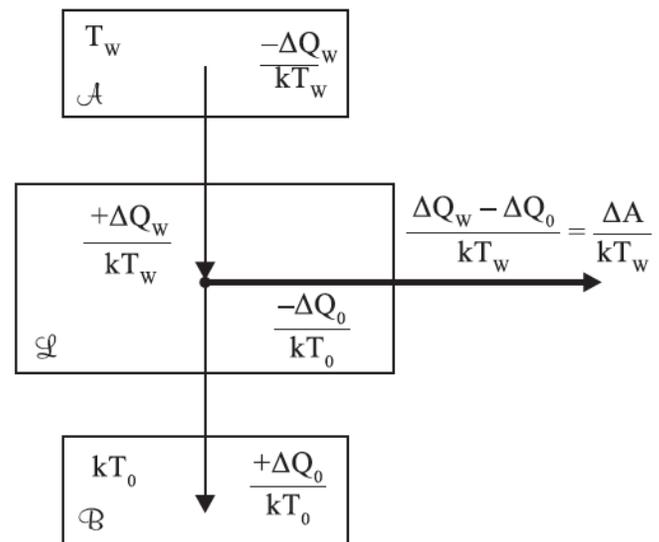


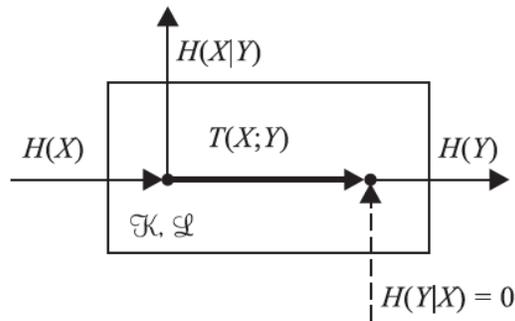
Figure 4: The schema of a reversible Carnot Cycle.

On the channel  $K$  the average information amounts  $H(X)$ ,  $H(Y)$ ,  $H(Y|X)$ , which have been expressed in (2.6) and (2.8), and with their values satisfying (5.1), are defined as:

$$H(X) \stackrel{\text{Def}}{=} \frac{\Delta Q_W}{kT_W} \left[ = \frac{\Delta Q_0}{kT_0}, \text{ following (4.3)} \right], \quad (5.2)$$

$$H(Y) \stackrel{\text{Def}}{=} \frac{\Delta A}{kT_W} = \frac{\Delta Q_W - \Delta Q_0}{kT_W} = \frac{\Delta Q_W}{kT_W} \cdot \eta_{\max} = H(X) \cdot \eta_{\max} \stackrel{\Delta}{=} \Delta I,$$

$$H(Y|X) \stackrel{\text{Def}}{=} 0.$$



**Figure 5:** The information transfer channel modelling a reversible Carnot Cycle.

Thus we assume the medium  $L$  going through a reversible Carnot cycle  $O$  works as a thermodynamic, average-value model of an information channel  $K$  ( $K \sim L$ ) and that the equation (2.12) is valid for the values of the quantities in (5.2).

Following from (4.3), the definition (5.2) and the assumption (11) we then have

$$\frac{\Delta Q_W}{kT_W} \cdot \eta_{\max} - 0 = \frac{\Delta Q_W}{kT_W} - H(X|Y) \text{ and thus} \quad (5.3)$$

$$H(X|Y) = \frac{\Delta Q_W}{kT_W} \cdot (1 - \eta_{\max}) = \frac{\Delta Q_W}{kT_W} \cdot \beta = \frac{\Delta Q_0}{kT_0}, \quad \beta = \frac{T_0}{T_W}.$$

Thus for the transinformation defined in (2.9), (2.10) and using the definitions (5.2), we have

$$T(X;Y) = H(X) \cdot (1 - \beta) = H(X) \cdot \eta_{\max}, \text{ then} \quad (5.4)$$

$$T(Y;X) = \frac{\Delta Q_W}{kT_W} \cdot \eta_{\max} = \frac{\Delta A}{kT_W} \text{ and thus } T(X;Y) = \Delta I = T(Y;X).$$

As a consequence of Thomson-Planck's formulation of the II. Principle of Thermodynamics the result (5.3) and the relations (5.4) imply the sharp inequality in the relation

$$\Delta I = T(X;Y) < H(X). \quad (5.5)$$

The equality in the relation (5.5) is valid for all reversible Carnot cycles (with temperatures  $T_W$  and  $T_0$ ) viewed informationally, and can be considered to be an *information* formulation of the first part of the Carnot's theorem.

When the medium  $L$  has gone through the cycle  $O$  once [and in agreement with (4.4)], we have the following for the change  $\Delta S_L$  of the heat entropy  $S_C$  of the whole reversible Carnot engine:

$$\Delta S_L = \oint_O \frac{\delta Q}{T} = \frac{\Delta Q_W}{kT_W} - \frac{\Delta Q_0}{kT_0} = 0. \quad (5.6)$$

As a consequence of the additivity of (substitute reversible [24]) changes of heat entropy, when the medium  $L$  has gone through the reversible cycle  $O$  once, we have the following for the change  $\Delta S_{AB}$  of the heat entropy  $S_C$  in the system ( $AB$ ) (consisting of the heater  $A$  and the cooler  $B$ ):

$$\Delta S_{AB} = -\frac{\Delta Q_0}{kT_0} + \frac{\Delta Q_W}{kT_0} = \frac{\Delta Q_0}{kT_0} \cdot \eta_{\max} = \frac{\Delta Q_W}{kT_W} \cdot \eta_{\max}. \quad (5.7)$$

A reversible Carnot engine is an isolated system, in which the transformation of heat energy  $\Delta Q_W \sim x$  to mechanical energy  $\Delta A \sim y$  is being performed. As a consequence of the additivity of (substitute reversible) changes of heat entropy, and when (5.6) and (5.7) are used, we have the following for the *result* change  $\Delta S_C$  of the heat entropy  $S_C$  of the whole reversible Carnot engine:

$$\Delta S_C = \Delta S_L + \Delta S_{AB} = \frac{\Delta Q_W}{kT_W} \cdot \eta_{\max}. \quad (5.8)$$

The derivation (5.4) and the equation (5.8) together then imply that

$$\Delta S_C - T(X;Y) = H(X) \cdot (\eta_{\max} - \eta_{\max}), \text{ then} \quad (5.9)$$

$$\Delta S_C - \Delta I = 0 \text{ or also } \Delta(S_C - I) = 0.$$

Therefore the result change  $\Delta S_C$  (5.8) of the heat entropy  $S_C$  of the whole reversible Carnot engine, together with the output information  $\Delta I$  defined in (5.2), satisfies **Brillouin's** [2, 20] **extended** formulation of the II. Principle of Thermodynamics<sup>7</sup>

$$\Delta(S_C - I) \geq 0 \text{ or also } d(S_C - I) \geq 0. \quad (5.10)$$

<sup>7</sup>The *information* member  $I$  does not exist in the traditional (differential) formulation of this theorem,  $dS \geq 0$ .

We assume the relation (5.9), incorporating the equality in (5.10), is valid for any information transfer, such as a *watching* process (i.e. transmitting, measuring, recording, computing, etc.), using energy transformations<sup>8</sup> expressible in a *reversible* thermodynamic way, such as a cycle  $O$  running in  $L$ .

This process can transfer a message  $x \sim \Delta Q_w$  from a source of messages with the information entropy  $H(X) = \frac{\Delta Q_w}{kT_w}$ . It is transmitting a message with the same average amount of information  $H(X)$ . It also could be measuring the status ( $x$ ) of the thermodynamic system  $A$  or  $(AB)$  observed through the process  $O$ . With an information transfer process of this type we obtain an output message  $y \sim \Delta A$  with the average amount of information  $\Delta I = H(Y) < H(X)$ . This information gain is expressed by the by-temperature-reduced output work  $\Delta A$  performed, while the system  $L$  underwent the cycle  $O$  once, at the temperature  $T_w$ . This expresses the definition (5.2) and the equation (2.12).

Consequently on receiving the output information  $\Delta I = \frac{\Delta A}{kT_w}$  in  $O$ , the *distinguishability* of the structure of the mechanical output of this cycle increases in volume, and is realized by the addition  $\Delta A$  to the potential output energy; the distinguishability being measured by  $\Delta I$ . But at the same time the *thermodynamic* distinguishability of the parts  $A$  and  $B$  of the system  $(AB)$ , the structure of which we express by the mutually different volumes of heats in  $A$  and  $B$ , has decreased in volume by exactly the value  $\Delta S_c$  (5.8). Thus we have  $\Delta S_c = \Delta I$ , stated in (5.9), and in (5.10) with the equals sign.

The increasing  $\Delta I$  of the *structural* distinguishability of the mechanical output models the receiver's average information increase by exactly the average information addition  $\Delta I$  contained in any message  $y$  on the output of  $K(\sim L)$ .

The value  $\Delta S_c$  in  $O$  thus represents the increase in the whole *extensity* (see remark 3) of energy used for coding the input message, and thus the indistinguishability of this energy within the whole isolated transfer chain. In agreement with (5.9) and

(5.10) we can say that any such *measuring influences what is measured*.

In the following section we will show that the measured subject [in our heat example  $A$  or  $(AB)$ ] is not only influenced by the way measuring is *organized* (here described by the cycle  $O$  with  $\Delta Q_{0,x} = 0$ ), but also that the result [ $y$  or  $H(Y)$ ] of measuring is influenced by its *construction* in a *real* environment (further described by the irreversible cycle  $O'$  with  $\Delta Q_{0,x} > 0$ ).

### 5.2. Irreversible Carnot Cycle and Noise Channel

We will now consider an irreversible Carnot cycle  $O'$  running in the medium  $L$  [its irreversibility caused by a non-ideal working medium  $L$  undergoing such a cycle with a finite, but not zero, speed (4.5) and (4.6)], to be a thermodynamic, average-value realization or, as such, as a model of an information transfer process running in a channel  $K$  *with noise*. For the noise information entropy  $H(Y|X)$  defined in (2.8) it is valid that  $H(Y|X) \neq 0$ . Thus we can assume that the medium  $L$  going through an irreversible cycle works as an average-value, thermodynamic realization, model, of an information channel  $K$  ( $K \sim L$ ).

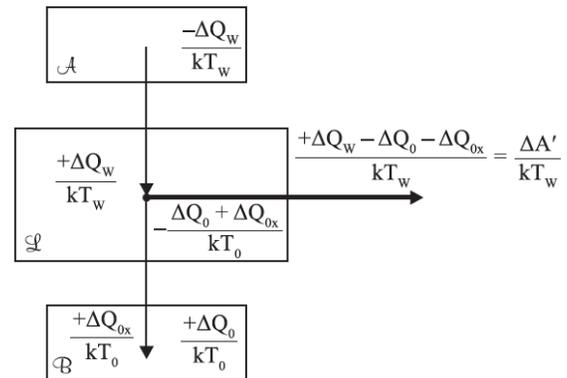


Figure 6: The schema of an irreversible Carnot Cycle.

On the channel  $K$  the average information amounts  $H(X)$ ,  $H(Y)$ ,  $H(X|Y)$ , which have been expressed in (2.6) and (2.7), and  $H(X)$ ,  $H(X|Y)$  with their values satisfying (5.1), (5.2) and (5.3), are defined as:

$$H(X) \stackrel{\text{Def}}{=} \frac{\Delta Q_w}{kT_w}, \tag{5.11}$$

$$H(Y) \stackrel{\text{Def}}{=} \frac{\Delta A'}{kT_w} = H(X) \cdot \eta_{max} - \frac{\Delta Q_{0,x}}{kT_w} = \frac{\Delta Q_w}{kT_w} \cdot \eta = H(X) \cdot \overset{\Delta}{\eta} = \Delta I'$$

and following the result (4), for the reversible case we define

$$H(X|Y) \stackrel{\text{Def}}{=} \frac{\Delta Q_0}{kT_w},$$

<sup>8</sup>Generally we could consider any type of so-called *directly-shared-energy* other than heat (e.g. electricity), and instead of the term *heat entropy* we could then speak about *extensity* and *Extensity Grow Principle* of the energy considered [6]. Naturally thermodynamic irreversibility would again reveal itself by the generation and dissipation of noise heat, i.e. on the electrical resistants in the case of electric energy

because, in respect to the mechanical work  $\Delta A'$  gained when  $L$  has gone through the cycle once, an irreversible Carnot cycle  $O'$  can be considered to be an additive superposition of its reversible part  $O$  [(4.3) is valid], and its irreversible part (with noise given by the production of the heat  $\Delta Q_{0,x} > 0$  in the medium  $L$ ). The noise heat is drained off into  $B$  within the "added" isothermal compression, (4.11) and (4.12).

We can assume that for the values of the quantities  $H(X)$ ,  $H(Y)$ ,  $H(X|Y)$ ,  $H(Y|X)$ , defined in (5.11), the equation (2.12) is valid and thus, similarly as in the previous subsection, we have

$$\frac{\Delta Q_w - \Delta Q_0 - \Delta Q_{0,x}}{kT_w} - H(Y|X) = \frac{\Delta Q_w}{kT_w} - \frac{\Delta Q_0}{kT_w} \text{ and consequently}$$

$$H(Y|X) = -\frac{\Delta Q_{0,x}}{kT_w} = -\frac{\Delta Q_{0,x}}{kT_0} \cdot \beta < 0, \beta = \frac{T_0}{T_w}, T_w \geq T_0 > 0. \quad (5.12)$$

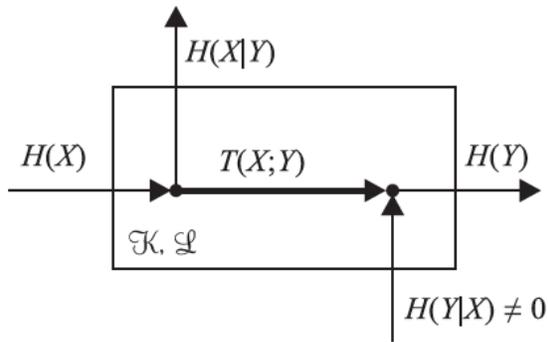


Figure 7: The information transfer channel modelling an irreversible Carnot Cycle.

In agreement with the definitions (5.11), the relations (5.12) for the average noise information  $H(Y|X)$  in the channel  $K(\sim L)$  are in accordance with the fact that noise heat  $\Delta Q_{0,x} > 0$  is inevitably drained off from the medium  $L$  (the sign  $-$ ) at the temperature  $T_0$ . This reduces the mechanical work  $\Delta A$  generated in the reversible part  $O$  of the cycle  $O'$  from the heat  $\Delta Q_w$  when the temperature of  $L$  is at  $T_w$ . This fact is expressed by the denominator of the first ratio in (5.12). The heat  $\Delta Q_{0,x}$  represents the above-mentioned construction *influence* on the definition (expressed thermodynamically by  $O$ ) of the measuring process itself.

It is obvious from (5.12), as well as from the equation (2.12), that the relations (5.11) describe a transfer channel  $K$  with negative values of noise added, and thus with the *subtractive* noise  $H(Y|X) < 0$ , which is caused by the draining off of the heat  $\Delta Q_{0,x}$  ( $-\Delta Q_{0,x} < 0$ ) from the medium  $L$ . With regards to the information loss  $H(X|Y)$  from the

transferred information  $H(X)$  in any input message  $x \sim \Delta Q_w$  within the reversible part ( $O$ ) of the irreversible ( $O'$ ), the heat  $\Delta Q_{0,x}$  is that noise which causes *another* (average) information loss  $H(Y|X)$  [the sign  $-$  in (5.12)] in  $O'$ .

For the transinformation value defined in (2.9), (2.10) and following the definitions (5.11) and (5.12), we have

$$T(X;Y) = \frac{\Delta Q_w}{kT_w} - \frac{\Delta Q_0}{kT_w} = \frac{\Delta Q_w}{kT_w} \cdot \eta_{max} \quad (5.13)$$

$$= H(X) \cdot \eta_{max} \text{ but also}$$

$$T(Y;X) = \frac{\Delta Q_w - \Delta Q_0 - \Delta Q_{0,x}}{kT_w} - \left(-\frac{\Delta Q_{0,x}}{kT_w}\right) = H(X) \cdot \eta_{max}.$$

The left sides of the equations in (5.13) are equal, and the transinformation satisfies the requirement of symmetry (2.12). Also it is obvious that the relations (5.13) are the same as those stated in (5.4) for a noiseless transfer.

The definitions (5.11), and the relations (5.13), together imply the following relations

$$\Delta I' = T(X;Y) \text{ when } \Delta Q_{0,x} = 0, \text{ then } \Delta I' = \Delta I, \quad (5.14)$$

$$\Delta I' < T(X;Y) \text{ when } \Delta Q_{0,x} > 0, \text{ then } \Delta I' < \Delta I.$$

Satisfying its definition, and with the given meaning of the quantities  $H(\cdot)$ ,  $H(\cdot|\cdot)$ , the transinformation  $T(X;Y)$  is the *maximal* (average) amount of output information that can be gained within either a reversible or irreversible Carnot cycle. In accordance with the second part of Carnot's theorem this is also true for *any other* heat cycle in the medium  $L$  with the extreme working temperatures  $T_w$  and  $T_0$ ,  $T_w \geq T_0 > 0$ . [These cycles are considered to be models of a transfer process with a given  $H(X)$  in a channel  $K(\sim L)$ .] The sharp inequality for the  $\Delta I'$  in (5.14) is a consequence of the second part of Carnot's theorem. Thus, following from (5.11) and assuming that (2.12) is valid, the inequalities

$$\Delta I' \leq T(X;Y) < H(X) \quad (5.15)$$

can be considered to be the *information* formulation of Kelvin's and Thomson-Planck's theorem and thus Carnot's theorem [the second part and also the first part; compare with (5.5) and (5.14)]. Therefore they are the *information* formulation of the II. Principle of Thermodynamics.

Because the value  $\eta_{max}$  is the maximum of a set of efficiencies  $\eta$ , it is obvious that  $T(X;Y) = T_{max}^\Delta(X;Y)$ .

And, because it satisfies the definition of information capacity as the maximum (supremum) of a set of transinformation, the transinformation  $T(X;Y)$ , stated in (5.13) and (5.4), is the capacity  $C_{T_W T_0}$  for the given extreme temperatures  $T_W$  and  $T_0$ . Consequently on the set of all heat engines with these extreme temperatures we define

$$C_{T_W T_0}^{\text{Def}} = T(X;Y), T(X;Y) = \frac{\Delta Q_W}{kT_W} \cdot \frac{T_W - T_0}{T_W}. \quad (5.16)$$

When the states of the medium  $L$  have gone through the irreversible cycle  $O'$  once, and in agreement with (4.12), we have the following for the change  $\Delta S_L$  of the heat entropy  $S_C$  of the whole irreversible Carnot engine:

$$\Delta S_L = \oint_{O'} \frac{\delta Q}{T} = \frac{\Delta Q_W}{kT_W} - \frac{\Delta Q_0 + \Delta Q_{0x}}{kT_0} = -\frac{\Delta Q_{0x}}{kT_0} < 0 \quad (5.17)$$

and following from (5.12), we can write this informationally

$$\Delta S_L = H(Y|X) \cdot \beta^{-1}, \beta = \frac{T_0}{T_W}, T_W \geq T_0.$$

When the medium  $L$  has gone through the irreversible cycle  $O'$  once, and due to the fact that heat entropy is additive (considering substitute changes [24] are reversible), we have the following for the change  $\Delta S_{AB}$  of the heat entropy  $S_C$  in the system  $(AB)$ :

$$\Delta S_{AB} = -\frac{\Delta Q_0}{kT_W} + \frac{\Delta Q_W}{kT_0} + \frac{\Delta Q_{0x}}{kT_0} = \frac{\Delta Q_W}{kT_W} \cdot \eta_{max} + \frac{\Delta Q_{0x}}{kT_0} \quad (5.18)$$

and, following from (5.12) and (5.13), written informationally

$$\Delta S_{AB} = T(X;Y) - H(Y|X) \cdot \beta^{-1} = T(X;Y) - \Delta S_L, \\ \beta = \frac{T_0}{T_W}, T_W \geq T_0.$$

The whole irreversible Carnot engine is an isolated system, in which the transformation of heat energy  $\Delta Q_W \sim x$  to mechanical energy  $\Delta A' \sim y$  is performed. As a consequence of the additivity of heat entropy when substitute changes are considered to be reversible, and when (5.17) and (5.18) are used, we have the following for the resulting change  $\Delta S_C = \Delta S_L + \Delta S_{AB}$  of the heat entropy  $S_C$  of the whole irreversible Carnot engine:

$$\Delta S_C = -\frac{\Delta Q_{0x}}{kT_0} + \left( \frac{\Delta Q_W}{kT_W} \cdot \eta_{max} + \frac{\Delta Q_{0x}}{kT_0} \right) = \frac{\Delta Q_W}{kT_W} \cdot \eta_{max} \quad (5.19)$$

and following (5.13) we can write informationally

$$\Delta S_C = H(X) \cdot \eta_{max} = T(X;Y) = C_{T_W T_0}.$$

The relation (5.19) for the result change  $\Delta S_C$  of the heat entropy  $S_C$  of the whole irreversible Carnot engine is the same as in the case of a noiseless transfer within a reversible Carnot engine (5.8). Following from (5.14) and (5.19) we can thus immediately derive

$$\Delta S_C - \Delta I' > 0 \text{ or also } \Delta(S_C - I') > 0, \quad (5.20)$$

[see (5.9)].

The inequality (5.20) reveals that the result change  $\Delta S_C$  of the heat entropy  $S_C$  of the whole irreversible Carnot engine stated in (5.19), together with the output average information  $\Delta I'$  defined in (5.11), satisfies Brillouin's [2] extended formulation of the II. Principle of Thermodynamics,

$$d(S_C - I') \geq 0 \text{ or, stated informationally,} \\ d[T(X;Y) - H(Y)] \geq 0. \quad (5.21)$$

It is obvious from both (5.11) and (5.19) that for the difference in (5.20) we have

$$\Delta S_C - \Delta I' = H(X) \cdot \eta_{max} - \left( H(X) \cdot \eta_{max} - \frac{\Delta Q_{0x}}{kT_W} \right) = \frac{\Delta Q_{0x}}{kT_W} > 0 \quad (5.22)$$

and, using the result (5.17), we can write informationally,

$$\Delta S_C - \Delta I' = |H(Y|X)| \cdot \beta = |\Delta S_L| \cdot \beta, \beta = \frac{T_0}{T_W}, T_W \geq T_0 > 0.$$

Thus the result change  $\Delta S_C$  of the heat entropy  $S_C$  of the whole irreversible Carnot engine is greater than the output information  $\Delta I'$  transferred through it. In this case the result of the information transfer is worse than the noiseless case by the difference (5.22). Following from (5.9) and (5.19) we can see that  $\Delta S_C = \Delta I = T(X;Y)$ .

When we use the definitions (5.2) and (5.11) directly, and express  $\eta$  from (4.9) using (4.7) and (4.8), it becomes obvious that we can state

$$\Delta I - \Delta I' = H(X) \cdot \left[ \eta_{max} - \left( \eta_{max} - \frac{\Delta Q_{0x}}{\Delta Q_W} \right) \right] \\ = \frac{\Delta Q_W}{kT_W} \cdot \frac{\Delta Q_{0x}}{\Delta Q_W} = \frac{\Delta Q_{0x}}{kT_W} > 0. \quad (5.23)$$

The equality of the left sides in (5.22) and (5.23) is obvious. Following from the definition (5.11), and with  $0 \leq \eta < \eta_{max} < 1$ , we can state

$$\Delta I' = \frac{\Delta Q_W}{kT_W} \cdot \eta_{max} - \frac{\Delta Q_{0x}}{kT_W} = \frac{\Delta Q_W}{kT_W} \cdot \eta_{max} - \frac{\Delta Q_{0x}}{kT_0} \cdot \beta \quad (5.24)$$

and following (5.17) we can write informationally

$$\begin{aligned} \Delta I' &= H(X) \cdot \eta_{max} + \Delta S_L \cdot \beta = C_{T_W T_0} + \Delta S_L \cdot \beta = \\ &= T(X;Y) - |\Delta S_L| \cdot \beta = T(X;Y) + \Delta S_L \cdot \beta, \quad \beta = \frac{T_0}{T_W}, T_W \geq T_0 > 0. \end{aligned}$$

The quantity  $|\Delta S_L|$  we term *production of heat entropy* in the medium  $L$  (within the irreversible cycle  $O'$ ). From the relations (5.24) it is clear that the change in the *structure* of the mechanical output (created by the positive addition of  $\Delta A'$  to the output potential energy) is expressed by the value of  $\Delta I'$ . This is less than  $\Delta I$  at the value  $|\Delta S_L| \cdot \beta$  in the reversible case.

Therefore *noise output* exists, draining off the transferred information from the maximum:

$$\Delta I = T(X;Y) = H(X) \cdot \eta_{max} \text{ to the value}$$

$$\Delta I' = H(X) \cdot \eta < T(X;Y), \text{ Carnot's theorem (the second part).}$$

In our case of carnotized information transfers running in what we can term *Carnot's transfer system* (i.e. Carnot's engine as a thermodynamic, average-value realization, or model, of a shannon transfer chain), we have

$$\Delta I' < T(X;Y), \text{ when } \eta < \eta_{max}, \Delta I' = H(X) \cdot \eta, \quad (5.25)$$

$$\Delta I' = \Delta I = T(X;Y), \text{ when } \eta = \eta_{max}, \Delta I' = H(X) \cdot \eta_{max}.$$

In both reversible and irreversible cases we can state

$$T(X;Y) = H(X) \cdot \eta_{max}, \text{ Carnot's theorem (the first part).} \quad (5.26)$$

Because the I. and the II. Principle of Thermodynamics hold, it is obvious that the *thorough* transfer of any input message  $x \sim \Delta Q_W$  with the (average) information value  $H(X)$ , expressed by

$$T(X;Y) = H(X), \quad (5.27)$$

is only the limit ( $\eta_{max} \rightarrow 1$ ), but not achievable in reality.

As in the reversible case, our heat transfer process completes with the addition of  $\Delta S_C$  to the whole thermodynamic entropy  $S_C$ , and for the average information  $\Delta I'$  gained from  $H(X)$ , we have

$$\Delta S_C \geq \Delta I' = H(X) \cdot \eta \geq 0 \text{ where } \Delta S_C = \Delta I = H(X) \cdot \eta_{max}. \quad (5.28)$$

The equality  $\Delta I' = \Delta I$  is valid only in a reversible transfer system in which no heat dissipation, generated from its non-ideal properties, exists, i.e. where  $\Delta Q_{0x} = 0$ .

Our thermodynamic-information derivation based on a heat cycle demonstrates the fact that it is impossible, in the type of channel considered, for the *bound* [2, 20] information contained in an input message to be transferred without its (average) loss. Such information transfer can be worsened only by heat dissipation of energy, which means by *noise* heat ( $\Delta Q_{0x} > 0$ ) generated by the irreversible processes in the channel [described by a transformer  $L$  of input heat, which has non-ideal properties (inner friction)]. Simultaneously the whole thermodynamic entropy of the extended isolated system in which this process is running increases, and maximum average value of the output transferred information diminishes.

### Summarizing Note

Even in the case of a *noiseless* but repeatable transfer of an input message within our "heat" type channel it is *impossible* to transfer this message *without* the average loss of the information it contains. This loss is the consequence of both the I. and II. Principle of Thermodynamics which determine energy transformation in an isolated system. The resulting entropy increase  $\Delta S_C$  within this process is caused by the requirement that the transfer be repeatable, or cyclical. Consequently the reduction, denoted as  $H(X|Y)$ , of the input average amount of information  $H(X)$ , is *inseparable* from such a repeatable process, being its *necessary* condition;

*repeatability of our information transfer*

$\Rightarrow$  (average) loss of information.

This is the condition for a cyclical transfer  $O$  [transforming input energy (coding  $x$ ) to output energy (coding  $y$ )] to function, physically as expressed by (4.4) and informationally as expressed by the inequality in (5.5),

$$T(X;Y) < H(X).$$

This relation is the necessary condition (expressed informationally) for our channel  $K$  to function repeatedly. Therefore our conclusion

$$H(Y) < H(X)$$

information form of Thomson-Planck's formulation of the II. Principle of Thermodynamics, and our conclusion

$$T(X;Y) = H(Y)$$

is the information variant of the first part of Carnot's theorem.

Assuming that  $H(Y|X) < 0$  [information variant of Kelvin's formulation of the II. Principle of Thermodynamics for irreversible cyclical transfer  $O'$  when the respective relation (4.12) is valid], we also have the information variant of the second part of Carnot's theorem

$$H(Y) < T(X;Y).$$

Following the last four relations we can derive the inequality

$$H(Y) \leq T(X;Y) < H(X),$$

which is the *complete information formulation* of the II. Principle of Thermodynamics in our model, introduced only in [10] and in this paper. Also it is the necessary condition of channel equality (2.12). This equality can be considered, as such, as a reasoning of the equivalence of various formulations of the II. Principle of Thermodynamics. Or, we can see (2.12) as the most general formulation of the II. Principle of Thermodynamics.

Consequently and, as the example, our model, in which the last formulation is valid, satisfies Shannon's coding theorem [3] for the case the channel capacity  $C < H(X)$ ; thus transfer errors are the inevitable consequence of the fact the transfer exists and thus, it is the physical reasoning of the *DPE, Data Processing Enequality* [3],  $H(Z) < H(Y) < H(X)$ , when a transfer ( $X \rightarrow Y \rightarrow Z$ ) is considered be "carnotized".

### 5.3. Reverse Reversible Carnot Cycle and Channel

Reverse reversible Carnot cycle works as a *heat pump* described in subsection 4.3. In this cycle, comprehended as a thermodynamic, average-value realization, or model of the transfer process in a channel  $K \cong L$ , which is transferring an (arbitrary) input message  $x \in X$  containing the average information amount  $H(X)$ , we use these symbols and denotations:

$\Delta Q_0$  the heat drained off from the cooler  $B$  within the isothermal expansion at  $T_0$ ,

$\Delta A$  the mechanical work delivered to the cycle by the compression phase at  $T_w$ ,

$\Delta Q_w$  the output heat delivered to the heater  $A$  by the isothermal phase at  $T_w$ .

Further, we define the values of changes of information entropies on the channel  $K \cong L$  (with an information transfer process being realized by this

cycle) by the changes of its physical entropies, for instance, in this way:

$$H(X) \stackrel{\text{Def}}{=} \frac{\Delta A}{kT_w}, \text{ input entropy,} \tag{5.29}$$

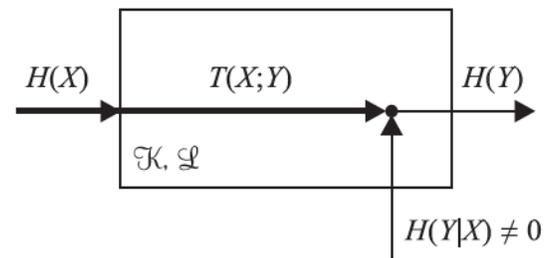
$\Delta A \cong x$  input message;

$$H(Y) \stackrel{\text{Def}}{=} \frac{\Delta Q_w}{kT_w} = \frac{\Delta Q_0 + \Delta A}{kT_w} = \Delta I, \text{ output entropy,}$$

$\Delta Q_w \cong y$  output message;

$$H(Y|X) \stackrel{\text{Def}}{=} \frac{\Delta Q_0}{kT_w} > 0, \text{ noise entropy,}$$

$\Delta Q_0$ , noise "message".



**Figure 8:** The information transfer channel modelling a reverse and reversible Carnot Cycle.

So, we consider a channel with *additive noise*. It is clear that

$$H(Y|X) = \frac{\Delta Q_0}{kT_w} \cdot \frac{T_0}{T_0} = \frac{\Delta Q_0}{kT_0} \cdot \beta = \frac{\Delta Q_w}{kT_w} \cdot \beta = H(Y) \cdot \beta, \beta = \frac{T_0}{T_w} \tag{5.30}$$

can be validated.

The noise with information entropy  $H(Y|X)$  is the integral part of the definition of the transfer information process. It is not generated by a positive production of the heat  $\Delta Q_{0,x} > 0$  in the working medium  $L$ .<sup>9</sup>

We are supposing further that for changes of information at the values  $H(X)$ ,  $H(Y|X)$ ,  $H(Y)$ ,  $H(X|Y)$  defined by (5.29) the relations (2.12) are valid and, then,

<sup>9</sup>From the relations for  $\eta$  and  $\eta_{max}$  it follows that  $\Delta Q_0 = f(T_0)$ , where function  $f(\cdot)$  is a not-negative function of the argument  $T_0$ ,  $f(T_0) \geq 0$ , for which  $\lim_{T_0 \rightarrow 0} f(T_0) = 0$  is valid.

$$\frac{\Delta A}{kT_w} - H(X|Y) = \frac{\Delta Q_0 + \Delta A}{kT_w} - \frac{\Delta Q_0}{kT_w} \quad (5.31)$$

and then

$$H(X|Y) \stackrel{\text{Def}}{=} 0.$$

So, we have a channel *without losses*. For transinformation  $T(X;Y)$ ,  $T(Y;X)$ , with respecting the definition (5.29) it is valid that

$$T(X;Y) = H(X) - H(X|Y) = \frac{\Delta A}{kT_w} - 0 = H(X) \quad (5.32)$$

and, also,

$$\begin{aligned} T(Y;X) &= H(Y) - H(Y|X) = \frac{\Delta Q_0 + \Delta A}{kT_w} - \frac{\Delta Q_0}{kT_w} \\ &= \frac{\Delta A}{kT_w} = H(X). \end{aligned} \quad (5.33)$$

The left sides of derivations (5.32) and (5.33) are then equal and they are valid for information amounts defined by (5.29) and for the result of derivation (5.31), the equation (2.12) for information conservation (both the average, information entropies and the "instantaneous" ones) in the channel too. (Within one run of the system  $L \cong K$  through the reverse Carnot cycle realizing a transfer process.)

It is evident that

$$\frac{H(X)}{H(Y)} = \frac{\frac{\Delta A}{kT_w}}{\frac{\Delta Q_w}{kT_w}} = \frac{\Delta A}{\Delta A + \Delta Q_0} = \eta_{\max} \quad (5.34)$$

and, consequently

$$H(X) = H(Y) \cdot \eta_{\max}, \quad (5.35)$$

where  $\eta_{\max}$  is the efficiency of the relevant direct cycle. So it is valid, in accordance with relations (5.30) and (5.33), that

$$T(X;Y) = H(Y) \cdot \eta_{\max}. \quad (5.36)$$

Now, let us notice the changes of thermodynamic entropy in an isolated system in which the described process is running:

$$\begin{aligned} \Delta S_{AB} &= \frac{-\Delta Q_0}{kT_0} + \frac{\Delta Q_0}{kT_w} = \frac{-\Delta Q_0}{T_0 T_w} \cdot \frac{T_w - T_0}{k} = \\ &= \frac{-\Delta Q_0}{kT_0} \cdot \eta_{\max} = -H(Y) \cdot \eta_{\max} < 0. \end{aligned} \quad (5.37)$$

Thermodynamic (Clausius) entropy  $S_{AB}$  of the system  $AB$  is then lowering its value - the (thermodynamic, heat) distinguishability of the systems  $A$  and  $B$  is growing. Of course, it is at a detriment of the mechanical work  $\Delta A$  delivered, or, of the entropy at the value  $\frac{\Delta A}{kT_w}$ . There is a need to gain this energy

(entropy) and it is possible within such an isolated system by an *unnatural* process of transformation heat to mechanical energy. But, this process is "running" at a background of a *natural* process of the heat transfer in accordance with the II. Principle of Thermodynamics.

We are considering such a reversible process, which is producing the mechanical work at a value  $\Delta A^* \geq \Delta A$  of functioning of both these cycles;

$$\begin{aligned} \frac{\Delta A^*}{kT_w^*} &= H(X^*) \cdot \eta_{\max}^* = H(X^*) \cdot \frac{T_w^* - T_0^*}{T_w^*} = \Delta S_{A^*B^*}, \\ T_w^* &\geq T_0^* > 0, \end{aligned} \quad (5.38)$$

$$\frac{\Delta A}{kT_w} = H(Y) \cdot \eta_{\max} = H(Y) \cdot \frac{T_w - T_0}{T_w} = -\Delta S_{AB}, \quad T_w \geq T_0 > 0.$$

For the whole change  $\Delta S$  of the entropy  $S$  of the whole isolated system in which both these processes are running, following the II. Principle of Thermodynamics, it is valid that

$$\Delta S = \Delta S_{A^*B^*} + \Delta S_{AB} \geq 0. \quad (5.39)$$

But, for  $\Delta S_{AB} \leq 0$ , it must be valid that

$$\Delta S_{A^*B^*} \geq |\Delta S_{AB}|. \quad (5.40)$$

This means that the decrease of entropy of the value  $|\Delta S_{AB}|$ , the greater addition of  $\Delta S_{A^*B^*}$  is to be generated, and, then, the whole entropy is growing at the value

$$\Delta S = \Delta S_{A^*B^*} - |\Delta S_{AB}| \geq 0. \quad (5.41)$$

The equality occurs when  $\eta_{\max}^* = \eta_{\max}$ . In another case  $\eta_{\max}^* > \eta_{\max}$  which, e.g. for  $T_0^* = T_0$ , means that  $\Delta Q_w^* > \Delta Q_w$  a  $T_w^* > T_w$ .

*The environment of the entropy decrease is being exhausted, as for its structure which is conserving energy, at a greater degree (or at least at the same one) - its undistinguishability, disorder (chaos) is growing more (or at the same degree) than this decrease, the local growing of the order or organisation, is.*

In the following parts of this text we will end our considerations about the analogy of heat transformation and information transfer. We will deal with direct processes with concentration of our attendance to their reversible type; the irreversible ones are expressible easily by those equivalent reverse ones.

However, we will return to the problem of a reverse cycle and its information-structural aspects in section 8 once more.

## 6. GIBBS PARADOX AND INFORMATION TRANSFER

We observe an equilibrium system  $A$ ,  $S^* = S_{\text{Claus}} = S_{\text{Boltz}} = -kNB^* = -kN \ln N$ .

Let, in accordance with the *solution* of Gibbs paradox, the integration constant  $S_0$  be the (change of) entropy  $\Delta S$ , added to the entropy  $\sigma$ , to figure out the measured entropy  $S_{\text{Claus}}$  of the equilibrium state of the system  $A$  (the final state of Gay-Lussac experiment) at a temperature  $\Theta$ . We have shown that without such correction, the lower entropy  $\sigma$  is evidenced,  $\sigma = S_{\text{Claus}} - \Delta S$ ,  $\Delta S = S_0$ .

Following the previous definitions and results we have

$$\Delta S = \frac{\Delta Q_0}{\Theta} = -nR \ln \frac{n}{\gamma}, \quad (6.1)$$

$$\ln \gamma = \frac{\Delta S}{knN_A} + \ln n = \frac{\Delta S}{kN} + \ln N - \ln N_A, \quad \gamma = N \Rightarrow \frac{\Delta S}{kN} = \ln N_A.$$

By the entropy  $\Delta S$  the "lost" heat  $\Delta Q_0$  (at the temperature  $\Theta$ ) is defined.

Thus, our observation can be understood as an information transfer  $T$  in an information channel  $K$  with entropies (2.6)-(2.8), but now bound; we have these information entropies per one particle of  $A$ :

$$\text{input } H(X) \stackrel{\text{Def}}{=} \frac{S^*}{kN} = \ln \gamma = -B^* = \ln N = -rB(r), \quad (6.2)$$

$$\text{output } H(Y) \stackrel{\text{Def}}{=} \frac{\sigma}{kN} = -B_{\text{Gibbs}} = -B_{\text{Boltz}} = -B(r),$$

$$\text{loss } H(X|Y) \stackrel{\text{Def}}{=} \frac{S_0}{kN},$$

$$\text{noise } H(Y|X) = 0 \quad \text{by (2.12);}$$

$$H(X|Y) = -rB(r) - [-B(r)] = B(r) \cdot (r-1) = (-B^*) \cdot \frac{r-1}{r}, \quad r \geq 1.$$

For a number  $m$  of cells of our railing in the volume  $V$  with  $A$ ,  $m \leq N$ , for a detail  $r$  of this our description of the "inner structure" of  $A$  (a thought structure of  $V$  with  $A$ ) and for the number  $q$  of diaphragmas creating our railing of cells and constructed in such a way that  $q \in \langle 1, m-1 \rangle$ , we have  $r = \frac{N-1}{q}$ .

The *maximal* detail of our "description", the accuracy of our observation of the system  $A$ , is achieved for  $r=1$ . Then  $B(r) = B^*$  and, for the output, the input and the loss information entropies, it is valid that

$$H(Y) = H(X) = -B^*, \quad H(X|Y) = 0.$$

The *minimal* accuracy, detail of our description of the observed system should be for  $r=\infty$ . In this case we should place  $q=0$  diaphragmas, no railing is laid and  $m=1$ . We are not considering the "inner structure" of  $A$  in this case. Thus we define an output information source  $Y$ , bound, for which  $H(Y) = -B_{\text{Gibbs}} = 0$ . Then, the result of such "observation" is 0, and the loss information entropy is

$$H(X|Y) = \frac{S^*}{kN} = \ln N = H(X).$$

Our observation of the equilibrium system  $A$ , including the *mathematical correction* for the Gibbs paradox, is then describable by the Shannon *scheme* (2.12), where

$$\begin{aligned} H(X) &= \frac{S_{\text{Claus}}}{kN}, \quad H(X|Y) = \frac{S_0}{kN}, \quad H(Y) \\ &= \frac{S_{\text{Claus}}}{kN}, \quad H(Y|X) = \frac{\Delta S}{kN}. \end{aligned} \quad (6.3)$$

However, a real observation process described in (6.3), equivalent to that one with  $r=1$ , is impossible [10].

We conclude this section by noting that the diminishing of the measured entropy value about  $\Delta S$  against  $S^*$  awaited, evidenced by the **Gibbs paradox**, **does not originate in an observed system itself**. Understood this way, **it is a contradiction of a gnozeologic character based on not respecting real properties of any observation** [10]. (The influence of our measuring, the properties of our measuring, are not included in our epistemology.)

## 7. OBSERVATION AND II. THERMODYNAMIC PRINCIPLE

The heat  $\Delta Q_0$  is defined by, and *defines* the loss entropy  $S_0 = \Delta S$  of our observation. We call it the *loss*

heat. It expresses an energy needed for a realization of our observation method. Thus, it is natural to consider it be "paid" by the observed system  $A$  and its driving off from  $A$  (with temperature  $\Theta$ ) to  $B$  (with temperature  $T_0 < \Theta$ ). Then,

$$\frac{\Delta Q_0}{T_0} = -\frac{Q - \Delta Q_0}{T_0} + \frac{Q}{T_0} = -\frac{Q - \Delta Q_0}{T_0} \cdot \frac{Q}{\Theta} + \frac{Q}{T_0} \cdot \frac{\Theta}{\Theta} = \frac{Q}{\Theta}. \quad (7.1)$$

Our observation method, a bound information transfer  $T$ , states the *identical* relation between two heats  $Q$  and  $\Delta Q_0$  at temperatures  $\Theta$  and  $T_0$  as a *reversible* (direct) Carnot cycle  $O$  with a *transformer*  $L$ , the working temperature  $T_0$  of its *cooler*  $B$  and  $\Theta$  of its *heater*  $A$ ; it supplies a mechanical energy  $\Delta A = Q - \Delta Q_0$ . Thus, the relation (1) represents the integral formulation of the II. P.T., written generally as

$$\sum_T \Delta S_{T \rightarrow 1} \stackrel{\Delta}{=} \oint_O dS = \oint_O \frac{\delta Q(\Theta)}{\Theta} = 0, \Theta \in (0, \infty), O = \bigcup_{\{t\}} O_{\{t\}}. \quad (7.2)$$

The cycle  $O$  is a thermodynamic *realization* of our observation, transfer  $T \equiv O$  [10]. The energy  $\Delta A$  bears the whole (average) information  $\sigma = S^* - S_0 = kN \cdot H(Y)$ , gained from  $A$  while it is observed by  $T$ .

We consider a system of stochastic quantities, a transfer channel  $K \equiv (X, Y)$ , with entropies  $H(X)$ ,  $H(Y)$ ,  $H(X|Y)$ ,  $H(Y|X)$  from (2.6)-(2.8).

In the realization  $O$  of  $T$ ,  $L \equiv K$ , the quantities  $X$ ,  $Y$ ,  $X|Y$ ,  $Y|X$  are bound, have a meaning of energies being input and being output from the transformer  $L$  undergoing  $O$ <sup>10</sup>. "Bound" equality (2.12), proved in information theory [3, 26], written generally as

$$\sum_T H_{\{t\}} \stackrel{\Delta}{=} \oint_T dH = \oint_O dB = 0, T = \bigcup_{\{t\}} T_{\{t\}}, \quad (7.3)$$

is valid for our  $T \equiv O$  or  $L \equiv K$  respectively, by the following assignment [10]:

$$H(X) = \frac{Q}{kN\Theta} = \frac{\Delta Q_0}{kNT_0}, H(Y) = \frac{\Delta A}{kN\Theta} = H(X) \cdot \eta_{max} \stackrel{\Delta}{=} \Delta I, \quad (7.4)$$

$$H(Y|X) = 0, H(X|Y) = \frac{Q}{kN\Theta} \cdot \beta = \frac{\Delta Q_0}{kN\Theta},$$

$$H(X) - H(X|Y) = H(X) \cdot \eta_{max} = \Delta I < H(X),$$

$$\text{where } \beta \stackrel{\Delta}{=} \frac{T_0}{\Theta} = \frac{\Delta Q_0}{Q}.$$

For changes of bound information entropies of systems  $L$ ,  $A$ ,  $B$  (defined by a realization  $O$  of our observation  $T$ ), and for the whole isolated system  $C$  in which our observation is performed, it is valid that [10],

$$\Delta S_L = \oint_O \frac{\delta Q}{k\Theta} = \frac{Q}{k\Theta} - \frac{\Delta Q_0}{kT_0} \stackrel{\Delta}{=} kN \cdot \Delta H_L = 0, \quad (7.5)$$

$$\Delta S_{AB} = -\frac{\Delta Q_0}{k\Theta} + \frac{\Delta Q_0}{kT_0} = \frac{\Delta Q_0}{kT_0} \cdot \eta_{max} \stackrel{\Delta}{=} kN \cdot \Delta H_{AB} = kN \cdot \Delta I \geq 0,$$

$$\Delta S_C = \Delta S_L + \Delta S_{AB} = \frac{Q}{k\Theta} \cdot \eta_{max} \stackrel{\Delta}{=} kN \cdot \Delta H_C = kN \cdot \Delta I \geq 0.$$

The last inequality for  $\Delta S_C$  states that the *extended* II. P.T. for reversible processes [2, 10] is valid within the system  $C$ .<sup>11</sup> Written informationally,

$$\oint_T dH = \Delta(H_C - I) = 0 \Rightarrow \Delta H_C = \Delta I \geq 0. \quad (7.6)$$

For the detail  $r \geq 1$ , for the entropy growth  $\Delta H_C$  and for the *efficiency*  $\eta_{max}$  of the realized definition of our observation method [10] it is, by (2.12), (7.4)-(7.6), valid that

$$\Delta H_C = -r \cdot B(r) - [-B(r) \cdot (r - 1)] = \Delta I = -B_{Gibbs}, \quad (7.7)$$

$$\Delta H_C = \frac{-B^*}{r} = \frac{-S^*}{kNr} = \frac{Q}{kNr\Theta}, \text{ and then}$$

$$\Delta S_C = \frac{Q}{r\Theta} \geq 0 \Rightarrow \eta_{max} = \frac{1}{r}, \beta = \frac{\Delta Q_0}{Q} = \frac{T_0}{\Theta} = \frac{r - 1}{r}, r \geq 1.$$

The value  $\eta_{max}$ , set by the accuracy  $r$  of our observation, is the coefficient of the entropy growth for any *natural* process of heat transition between hot and cool environments. It is the efficiency of a cyclic reversible transformation using this transition. The value  $1 - \eta_{max}$  sets the (average) information loss connected necessarily with an information transfer realized this way.

The definition  $\eta_{max} \stackrel{\text{Def}}{=} \frac{Q - \Delta Q_0}{Q} \in (0, 1)$  is a formulation of the *I.P.T.* too.

But, by derivations (7.4)-(7.7) it is clear that

<sup>10</sup>Thus, our observing method includes, "in itself", its own repeatability [10]

<sup>11</sup>The same growth of entropy  $S_C$  could occur in an irreversible case too. Heat  $\Delta Q_{0x} > 0$ , from irreversibility of our (realized) observation, measuring, would diminish output  $H(Y)$  from  $\Delta I$  to  $\Delta I - \frac{\Delta Q_{0x}}{kN\Theta}$ ;  $\Delta H_C = \Delta I \geq 0$  any way [10].

$$\eta_{max} = \frac{\Delta H_C}{H(X)}, \quad (7.8)$$

$$H(X) > 0 \Rightarrow \Delta H_C \geq 0, \Delta S_C = kN \cdot \Delta H_C \geq 0 \quad [II.P.T.].$$

Thus, in (7.8) the equivalent definition of  $\eta_{max}$  is given and it provides a formulation of the II. P.T. too. By (7.8) **the common formulation of the Principle of Equivalence of the I. and the II. P.T. is stated.**

Thus, by the derivation of (2.12) and by considering its equivalence with the information description of a reversible heat cycle [10], **the II. P.T. is proved.**

### 7.1. Equivalence Principle of Thermodynamics

Even more, the III. P.T. is deducible from (7.8) in that the interval of values of  $\eta_{max}$  must be, due to validity of the I. and the II. P.T., open on its left side  $\Rightarrow T_0 > 0^\circ \text{ K}$ .

At the end, let us formulate the theorem, the *Principle of Equivalence of the I., the II. and the III. P.T.*, the proof of which is provided by this paper.

#### Theorem (Equivalence Principle of Thermodynamics).

Let  $(X,Y)$  be a system of (bound) stochastic quantities with information entropies  $H(X)$ ,  $H(Y)$ ,  $H(X|Y)$ ,  $H(Y|X)$  or, with relevant thermodynamic entropies  $S_{[-1]} = (Q_{[-1]}) \cdot \Theta^{-1} = kN \cdot H(\cdot)$ .

Let  $H(X|Y) = \frac{\Delta Q_0}{kN\Theta}$ ,  $\Delta Q_0 = Q \cdot \frac{T_0}{\Theta} = kN \cdot H(X) \cdot T_0$  where  $\Theta \geq T_0 > 0$  and  $\Theta^{-1}$  is Pfaffian integrating factor for  $Q_{[-1]}$ ,  $N$  is a number of particles of  $A \cong X$  (the selecting space of  $X = [X, p(\cdot)]$ ).

Then, by (2.12) with  $H(Y|X) = 0$  and for  $S^* = kN \cdot H(X)$ ,  $S_0 = (S^*) \cdot \frac{T_0}{\Theta}$ ,

$$\frac{Q - \Delta Q_0}{Q} = \eta_{max} \in (0,1), \quad [I.P.T.] \Leftrightarrow \quad (7.9)$$

$$\Leftrightarrow \frac{(S^*) - S_0}{S^*} = \frac{H(X) - H(X|Y)}{H(X)} = \eta_{max} \Rightarrow$$

$$\Rightarrow (S^*) - S_0 = \Delta S_C \geq 0 \quad \text{and}$$

$$H(X) - H(X|Y) = \Delta H_C \geq 0, \quad [II.P.T.] \Leftrightarrow$$

$$\Leftrightarrow \frac{\Theta - T_0}{\Theta} = \eta_{max} \Rightarrow \Theta \geq T_0 > 0, \quad [III.P.T.].$$

The II. P.T. is *derivable* logically by properties of a (bound) stochastic system  $(X,Y)$ ; the others are derivable from this one; they all are equivalent.<sup>12</sup> *Q.E.D.*

## 8. RESULTS AND DISCUSSION

Let us think now, in a *free way* about possible applications of the results of this article in *biology*.<sup>13</sup> We can consider a principle loss of information (structure) within the process of proliferation of cells by their duplication (dividing). Within any dividing of a cell (*the predecessor*) its *follower (equivocant)* is generated and the a distortion of the *duplicated (copied) structure (information)* of the paren cell arises. This loss is measurable by the quantity of (average) information amount. It is a loss of part of message being copied (transferred), a loss of information within this process of such an information transfer. The whole structure of the cell is that message, including "a program" for its functionality in a texture.

The result of this is that the continuously decreasing precise of the structure of the followers is generated in a sequence of copying, dividing the cells, and, the whole biological organism, the collection of textures of cells, is aging by "the tooth of time" - by the loss of their *structure*, precision of the construction of the follower cells (for this, by the loss of both their inner and outer bounds). The notion of structure of an object (a message, a cell) is to be understood as follows: The object has more structure with the greater amount of information it contains, the more complicated it is - the greater number of parts bound mutually it contains, and, also, the less probability of its construction, of its structure is evidenced, or, the less stability as an isolated physical system it reveals. At the end, following a number of divisions, the incompatibility of the resulting structure of the last cell (the incompatibility of the information amount being represented by this cell) with such a minimal structure (information), which maintains its ability of both inner and outer

<sup>12</sup>Our paper is arranged as a sequence of successive steps, definitions and derivations (2.4)-(2.12) and interpretations (7.3)-(7.7) especially, stating gradually, the relation between combinatorial definition of Shannon (information) entropy and Boltzman and Clausius (thermodynamic) entropy, and, finally, resulting in (7.8) and (7.9). Although we combined the known facts about heat transformations and the Shannon's concept of an information transfer chain, this combination presented has not been used yet, as far as the author of the paper is informed.

A next possible application of the opinion presented could be used for another transfer (transmission, transition) structures, e.g. electric circuits or computational processes and, also it could be used for stating of limits of possible behavior of anticipatory systems of various types, e.g. for dynamic systems with their bifurcations viewed as such a kind of information transfer process. The achieved results are planned to be used in studying relations of Thermodynamics and Logics and Thermodynamics of Computing.

<sup>13</sup>The author is emphasizing strongly that the ideas expressed below are *free hypotheses* of a functional analogy type only.

communication in such a way that it is recognizable as "a certain cell" of the certain cell type - so as it is being alive, arises.

This mechanism can be described in a functional way by our *direct* "carnotized" model of information transfer. Within any run of our model Carnot cycle the less output (average) information is gained in comparison with the input one. If this result information is used again less information is generated etc. But, contemporarily, any run of the cycle generates the positive addition of entropy (thermodynamic) of a wider isolated system in which this transformation (transfer) runs. In this model it is the less and less distinguishability of parts of the whole model heat engine - our thermodynamic model of that aging by "a tooth of time". So, the sought "gen of aging" could be nothing more than a datum about the precision of that duplication; in our model it is the efficiency of the transformation of the input energy. So it is clear that the loss of information transferred this way is a suitable functional model for the case of the dividing of cells. As for cells, this principle reveals and signalizes itself by the phenomenon of the *shortening of telomeras* [Wen-Chi Hsueh, University of California, San Francisco; Proceedings of the National Academy of Sciences, 2007].

Within the pathological proliferation of cells the opposite situation arises. In this case the cells with a precise structure (but, by another type in comparison with the normal structure of its own original and "normal type" is). But, this growth of structure in a certain locality in the whole organism is paid by exhausting the energy from an environment of this locality. This environment is then in a lack of energy for its normal functionality; this energy is consumed in favor of this local growth of structure signalized by that phenomenon that the followers of the predecessors have the telomeras lengthened [Wen-Chi Hsueh].

This situation is describable again by our information-thermodynamic model, but by a *reverse* one this time. In this model a *local* decreasing of entropy is evidenced and then, the growth of structure of such a locality (within a wider isolated system) appears. However, for this decrease of entropy (for this growth of structure) the energy is needed, but, delivered from the environment of this locality, which leads to a growth of entropy of this environment and, consequently, to a growth of entropy of the whole isolated wider system (organism) - which is the price for the greater and greater structure of its (open) part - the problematic locality. In the case of cells we can see the wasting away of the whole organism. Our reverse information-thermodynamic model authorizes us to the awaiting of a higher stable (moderate, in average) body temperature of a patient and, also, a lower temperature of the problematic texture. Also, as for therapy, we are

authorized to consider energy consumption from the organism, the body as a whole, e.g. by its cooling (therapy by intensive freezing, swimming in a cold bath; especially in an inoperable cases of this disease). This could be the way to limit, at least for a short time, the positive flow of heat supporting the cell metabolic processes in problematic localities.<sup>14 15</sup>

So it seems to be right to await the notion that our information-thermodynamic opinion is viable, within the quantitative-functional-analogy point of view, to define the properties and quality of a wide variety of phenomena.

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<sup>14</sup>This means the lowering of the temperature gradient between the sound and the ill textures.

<sup>15</sup>For the pathology proliferating texture is consuming the energy from the rest of the organism it is logical to try to cut off this energy supply. From this point of view, the therapy by *devitalization* of a pathology tumour [MUDr. Karel Fortýn, CSc., Ústav živočišné fyziologie a genetiky AVČR v Libčechově, 1957, 1971-2001] seems to be a good method.

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Received on 25-06-2019

Accepted on 31-07-2019

Published on 23-08-2019

<https://doi.org/10.29169/1927-5129.2019.15.02>

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